## Thermal Effect on Pump Performance

Changes in the temperature of the liquid being pumped will clearly affect the vapor pressure,  $p_V$ , and



Figure 1: Typical cavitation performance characteristics for a centrifugal pump pumping water at various temperatures as indicated (Arndt 1981 from Chivers 1969).

therefore the NPSH or cavitation number. This effect has, of course, already been incorporated in the analysis or presentation of the performance by using the difference between the inlet pressure and the vapor pressure rather than the absolute value of the inlet pressure as a flow parameter. But there is another effect of the liquid temperature which is not so obvious and requires some discussion and analysis. It is illustrated by figure 1 which includes cavitation performance data for a centrifugal pump (Arndt 1981 from Chivers 1969) operating with water at different inlet temperatures. Note that the cavitation breakdown decreases substantially with increasing temperature. Somewhat counter-intuitively the performance actually improves as the temperature gets greater! The variation of the breakdown cavitation number,  $\sigma_b$ , with the inlet temperature in Chivers' (1969) experiments is shown in figure 2 which includes data for two different speeds and shows a consistent decrease in  $\sigma_b$  with increasing temperature. The data for the two speeds deviate somewhat at the lowest temperatures. To illustrate that the thermal effect occurs in other liquids and in other kinds of pumps, we include in figure 3 data reported by Gross (1973) from tests of the Saturn J-2 liquid oxygen inducer pump. This shows the same pattern manifest in figure 2. Other data of this kind has been obtained by Stepanoff (1961), Spraker (1965) and Salemann (1959) for a variety of other liquids.

The explanation for this effect is most readily given by making reference to traveling bubble cavitation though it can be extended to other forms of cavitation. However, for simplicity, consider a single bubble (or nucleus) which begins to grow when it enters a region of low pressure. Liquid on the surface of the bubble will vaporize to provide the increase in volume of vapor filling the bubble. Consider, now, what happens at two different temperatures, one "high" and one "low." At "low" temperatures the density of the saturated vapor is low and, therefore, the mass rate of evaporation of liquid needed is small. Consequently, the rate at which heat is needed as latent heat to effect this vaporization is low. Since the heat will be conducted from the bulk of the liquid and since the rate of heat transfer is small, this means that the amount by which the temperature of the interface falls below the bulk liquid temperature is also small. Consequently the vapor pressure in the cavity only falls slightly below the value of the vapor pressure at the bulk liquid



Figure 2: Thermodynamic effect on cavitation breakdown for a commercial centrifugal pump (data from Chivers 1969).



Figure 3: Effect of temperature on the cavitation performance of the J-2 liquid oxygen inducer pump (adapted from Gross 1973).

temperature. Therefore, the driving force behind the bubble growth, namely the difference between the internal pressure (vapor pressure) and the pressure far from the bubble, is not much influenced by thermal effects.

Now, consider the same phenomenon occuring at the "high" temperature. Since the vapor density can be many orders of magnitude larger than at the "low" temperature, the mass rate of evaporation for the same volume growth rate is much larger. Thus the heat which must be conducted to the interface is much larger which means that a substantial thermal boundary layer builds up in the liquid at the interface. This causes the temperature in the bubble to fall well below that of the bulk liquid and this, in turn, means that the vapor pressure within the bubble is much lower than otherwise might be expected. Consequently, the driving force behind the bubble growth is reduced. This reduction in the rate of bubble growth due to thermal effects is the origin of the thermal effect on the cavitation performance in pumps. Since the cavitation head loss is primarily due to disruption of the flow by volumes of vapor growing and collapsing within the pump, any reduction in the rate of bubble growth will lessen the disruption and result in improved performance.

This thermal effect can be extended to attached or blade cavities with only minor changes in the details.

At the downstream end of a blade cavity, vapor is entrained by the flow at a certain volume rate which will depend on the flow velocity and other geometric parameters. At higher temperatures this implies a larger rate of entrainment of mass of vapor due to the larger vapor density. Since vaporization to balance this entrainment is occurring over the surface of the cavity, this implies a larger temperature difference at the higher temperature. And this implies a lower vapor pressure in the cavity than might otherwise be expected and hence a larger "effective" cavitation number. Consequently the cavitation performance is improved at the higher temperature.

Both empirical and theoretical arguments have been put forward in attempts to quantify these thermal effects. We shall begin with the theoretical arguments put forward by Ruggeri and Moore (1969) and by Brennen (1973). These explicitly apply to bubble cavitation and proceed as follows.

At the beginning of bubble growth, the rate of growth rapidly approaches the value given by equation (Mbew8) and the important  $(dR/dt)^2$  term in the Rayleigh-Plesset equation (Mbew1) is roughly constant. On the other hand, the thermal term,  $\Theta$ , which is initially zero, will grow like  $t^{\frac{1}{2}}$  according to equation (Mbew6). Consequently there will be a critical time,  $t_C$ , at which the thermal term,  $\Theta$ , will approach the magnitude of  $(p_B(T_{\infty}) - p)/\rho_L$  and begin to reduce the rate of growth. Using the expression (Mbew6), this critical time is given by

$$t_C \approx \left(p_B - p\right) / \rho_L \Sigma^2 \tag{Mber1}$$

For  $t \ll t_C$ , the dominant terms in equation (Mbew1) are  $(p_B - p)/\rho_L$  and  $(dR/dt)^2$  and the bubble growth rate is as given by equation (Mbew8). For  $t \gg t_C$ , the dominant terms in equation (Mbew1) become  $(p_B - p)/\rho_L$  and  $\Theta$  so that, using equations (Mbew4) and (Mbew5), the bubble growth rate becomes

$$\frac{dR}{dt} = \frac{c_{PL} \left(T_{\infty} - T_B(t)\right)}{\mathcal{L}} \left(\frac{\alpha_L}{t}\right)^{\frac{1}{2}}$$
(Mber2)

which is typical of the expressions for the bubble growth rate in boiling. Now consider a nucleus or bubble passing through the pump which it will do in a time of order  $1/\Omega\phi$ . It follows that if  $\Omega\phi \gg 1/t_C$  then the bubble growth will not be inhibited by thermal effects and explosive cavitating bubble growth will occur with the potential of causing substantial disruption of the flow and degradation of pump performance. On the other hand, if  $\Omega\phi \ll 1/t_C$ , most of the bubble growth will be thermally inhibited and the cavitation performance will be much improved.

To calculate  $t_C$  we need values of  $\Sigma$  which by its definition (equation (Mbew7)) is a function only of the liquid temperature. Typical values of  $\Sigma$  for a variety of liquids are presented in figure 4 as a function of temperature (the ratio of temperature to critical temperature is used in order to show all the fluids on the same graph). Note that the large changes in the value of  $\Sigma$  are caused primarily by the change in the vapor density with temperature.

As an example, consider a cavitating flow of water in which the tension,  $(p_B - p)$ , is of the order of  $10^4 kg/m \ s^2$  or 0.1bar. Then, since water at  $20^{\circ}C$  has a value of  $\Sigma$  of about  $1m/s^{\frac{3}{2}}$ , the value of  $t_C$  is of the order of 10s. Thus, in virtually all pumps,  $\Omega\phi$  will be much greater than  $1/t_C$  and no thermal effect will occur. On the other hand at  $100^{\circ}C$ , the value of  $\Sigma$  for water is about  $10^3m/s^{\frac{3}{2}}$  and it follows that  $t_C = 10\mu s$ . Thus in virtually all cases  $\Omega\phi \ll 1/t_C$  and a strong thermal effect can be expected. In fact, in a given application there will exist a "critical" temperature above which one should expect a thermal effect on cavitation. For a water pump rotating at 3000rpm this "critical" temperature is about  $70^{\circ}C$ , a value which is consistent with the experimental measurements of pump performance.

The principal difficulty with the above approach is in finding some way to evaluate the tension,  $p_B-p$ , for use in equation (Mber1) in order to calculate  $t_C$ . Alternatively, the experimental data could be examined for guidance in establishing a criterion based on the above model. To do so equation (Mber1) is rewritten in terms of dimensionless groups as follows:

$$\Omega \phi t_C = \frac{1}{2} \left\{ \frac{p_B - p}{\frac{1}{2} \rho_L \Omega^2 R_T^2} \right\} \left\{ \frac{R_T^2 \Omega^3 \phi}{\Sigma^2} \right\}$$
(Mber3)



Figure 4: Thermodynamic parameter,  $\Sigma$ , as a function of temperature for various saturated fluids.

where the expression in the first curly brackets on the right hand side could be further approximated by  $(-C_{pmin} - \sigma)$  where  $C_{pmin}$  is a characteristic minimum pressure coefficient. It follows that the borderline between a flow which is broken down due to cavitation in the absence of thermal effects and a flow which is not broken down due to a beneficial thermal effect occurs when the ratio of times,  $\Omega \phi t_C$ , takes some critical value which we will denote by  $\beta$ . Equation (Mber3) with  $\Omega \phi t_C$  set equal to  $\beta$  would then define a critical breakdown cavitation number,  $\sigma_x$  ( $\sigma_a$  or  $\sigma_b$ ) as follows:

$$\sigma_x = -C_{pmin} - 2\beta \frac{\Sigma^2}{R_T^2 \Omega^3 \phi} \tag{Mber4}$$

The value of  $\sigma_x$  in the absence of thermal effects should then be  $(\sigma_x)_0 = -C_{pmin}$  and equation (Mber4) can be presented in the form

$$\frac{\sigma_x}{(\sigma_x)_0} = 1 - 2\beta \frac{\Sigma^2}{R_T^2 \Omega^3 \phi(\sigma_x)_0}$$
(Mber5)

It would then follow that the ratio of critical cavitation numbers,  $\sigma_x/(\sigma_x)_0$ , should be a simple function of the modified thermal effect parameter,  $\Sigma^*$ , defined by

$$\Sigma^* = \Sigma / \left\{ R_T^2 \Omega^3 \phi(\sigma_x)_0 \right\}^{\frac{1}{2}}$$
(Mber6)

To test this hypothesis, data from a range of different experiments is presented in figure 5. It can be seen that the data correspond very roughly to some kind of common curve for all these different pumps and liquids. The solid line corresponds to equation (Mber5) with an arbitrarily chosen value of  $\beta = 5 \times 10^{-6}$ . Consequently this attempt to model the thermal effect has succeeded to a limited degree. It should however be noted that the horizontal scatter in the data in figure 5 is more than a decade, though such scatter may be inevitable given the range of impeller geometries. We have also omitted one set of data, namely that of Chivers (1969), since it lies well to the left of the data included in the figure.



Figure 5: The ratio of the critical cavitation number  $\sigma_x$  ( $\sigma_a$  or  $\sigma_b$ ) to ( $\sigma_x$ )<sub>0</sub> (the value of  $\sigma_x$  in the absence of any thermal effect) as a function of the thermal effect parameter,  $\Sigma^*$ . Data is shown for a variety of pumps and liquids.



Figure 6: The parameter, B' (in  $m^{-1}$ ), for several liquids and  $H_T$  (in m) for centrifugal pumps operating at design flow rate and at a 3% head drop (adapted from Knapp *et al.* 1970 and Stepanoff 1964).

A number of purely empirical approaches to the same problem have been suggested in the past. All these empirical methods seek to predict the change in NPSH, say  $\Delta NPSH$ , due to the thermal effect. This quantity  $\Delta NPSH$  is the increment by which the cavitation performance characteristic would be

shifted to the left as a result of the thermal effect. The method suggested by Stahl and Stepanoff (1956) and Stepanoff (1961, 1964) is widely used; it is based on the premise that the cavitation characteristic of a particular pump operating at a particular speed with two different liquids (or with two different temperatures in the same liquid) would be horizontally shifted by

$$\Delta NPSH = H_{T1} - H_{T2} \tag{Mber7}$$

where the quantities  $H_{T1}$  and  $H_{T2}$  only depend on the thermodynamic properties of the two individual fluids considered separately. This generic property is denoted by  $H_T$ . For convenience, Stepanoff also uses the symbol B' to denote the group  $\rho_L^2 c_{PL} T_{\infty} / \rho_V^2 \mathcal{L}^2$  which also occurs in  $\Sigma$  and almost all analyses of the thermal effect. Then, by examining data from a number of single-stage 3500 rpm pumps, Stahl and Stepanoff arrived at an empirical relation between  $H_T$  and the thermodynamic properties of the following form:

$$H_T$$
 (in  $m$ ) = 28.9 $\rho_L g/p_V(B')^{\frac{4}{3}}$  (Mber8)

where  $p_V/g\rho_L$  is the vapor pressure head (in *m*) and *B'* is in  $m^{-1}$ . This relation is presented graphically in figure 6. Clearly equation (Mber8) (or figure 6) can be used to find  $H_T$  for the desired liquid and operating temperature and for the reference liquid at the reference operating temperature. Then the cavitation performance under the desired conditions can be obtained by application of the shift given by equation (Mber7) to the known cavitation performance under the reference operating temperature.