## Analyses of Cavitation in Pumps

In this and the sections which follow we shall try to give a brief overview of the various kinds of models which have been developed for the analysis of developed cavitation in a pump. Clearly different types of cavitation require different analytical models. We begin in this section with the various attempts which have been made to model traveling bubble cavitation in a pump and to extract from such a model information regarding the damage potential, noise or performance decrement caused by that cavitation. In a later section we shall outline the methods developed for attached blade cavitation. As for the other types of cavitation which are typically associated with the secondary flows (*e.g.*, tip vortex cavitation, backflow cavitation) there is little that can be added to what has already been described in the last chapter. Much remains to be understood concerning secondary flow cavitation, perhaps because some of the more important effects involve highly unsteady and transient cavitation.

To return to a general discussion of traveling bubble cavitation, it is clear that, given the pressure and velocity distribution along a particular streamline in a reference frame fixed in the impeller, one can input that information into the Rayleigh-Plesset equation (Mbew1) as discussed in section (Mbew). The equation can then be integrated to find the size of the bubble at each point along its trajectory (see examples in section (Mbew)). Such programs are equally applicable to two-phase flows or to two-component gas/liquid flows.

Since the first applications of the Rayleigh-Plesset equation to traveling bubble cavitation by Plesset (1949) and Parkin (1952) there have been many such investigations, most of which are reviewed by Holl (1969). A notable example is the work of Johnson and Hsieh (1966) who included the motion of the bubble relative to the liquid and demonstrated the possibility of some screening effects because of the motion of the bubble solutions of this type for flows around simple headforms the same programs can readily be used for the flow around a pump blade provided the pressure distributions on streamlines are known either from an analytic or numerical solution or from experimental measurements of the flow in the absence of cavitation. Such investigations would allow one to examine both the location and intensity of bubble collapse in order to learn more about the potential for cavitation damage.

These methods, however, have some serious limitations. First, the Rayleigh-Plesset equation is only valid for spherical bubbles and collapsing bubbles lose their spherical symmetry as discussed in section (Mbex). Consequently any investigation of damage requires considerations beyond those of the Rayleigh-Plesset equation. Secondly, the analysis described above assumes that the concentration of bubbles is sufficiently small so that bubbles do not interact and are not sufficiently numerous to change the flow field from that for non-cavitating flow. This means that they are of little value in predicting the effect of cavitation on pump performance since such an effect implies interactions between the bubbles and the flow field.

It follows that to model the performance loss due to traveling bubble cavitation one must use a twophase or two-component flow model which implicitly includes interaction between the bubbles and the liquid flow field. One of the first models of this kind was investigated by Cooper (1967) and there have been a number of similar investigations for two-component flows in pumps, for example, that by Rohatgi (1978). While these investigations are useful, they are subject to serious limitations. In particular, they assume that the two-phase mixture is in thermodynamic equilibrium. Such is certainly not the case in cavitation flows where an expression like the Rayleigh-Plesset equation is needed to describe the dynamics of disequilibrium. Nevertheless the models of Cooper and others have value as the first coherent attempts to evaluate the effects of traveling bubble cavitation on pump performance.

Rather than the assumption of thermodynamic equilibrium, two-phase bubble flow models need to

be developed in which the bubble dynamics are included through appropriate use of the Rayleigh-Plesset equation. In recent years a number of investigators have employed such models to investigate the dynamics and acoustics of clouds of cavitation bubbles in which the bubbles and the flow interact (see, for example, Chahine 1982, d'Agostino and Brennen 1983, 1989, d'Agostino *et al.* 1988, Biesheuval and van Wijngaarden 1984, Omta 1987). Among other things these investigations demonstrate that a cloud of bubbles has a set of natural frequencies of its own, separate from (but related to) the bubble natural frequency and that the bubble and flow interaction effects become important when the order of magnitude of the parameter  $\alpha A^2/R^2$  exceeds unity where  $\alpha$  is the void fraction and A and R are the dimensions of the cloud and bubbles respectively. These more appropriate models for travelling bubble cavitation have not, as yet, been used to investigate cavitation effects in pumps.

Several other concepts should be mentioned before we leave the subject of bubbly cavitation in pumps. One such concept which has not received the attention it deserves was put forward by Jakobsen (1964). He attempted to merge the free streamline models (which are discussed later in section (Mbes)) with his observations that attached cavities on the suction surfaces of impeller blades tend to break up into bubbly mixtures near the closure or reattachment point of the attached cavity. Jakobsen suggested that condensation shocks occur in this bubbly mixture and constitute a mechanism for head breakdown.

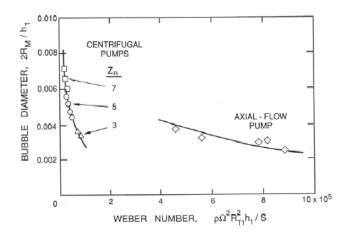


Figure 1: The bubble diameters observed in the blade passages of centrifugal and axial flow pumps as a function of Weber number (adapted from Murakami and Minemura 1978).

There are also a number of results and ideas that emerge from studies of the pumping of bubbly gas/liquid mixtures. One of the most important of these is found in the measurements of bubble size made by Murakami and Minemura (1977, 1978). It transpires that, in most practical pumping situations, the turbulence and shear at inlet tend to break up all the gas bubbles larger than a certain size during entry to the blade passages. The ratio of the force tending to cause fission to the surface tension, S, which tends to resist fission will be a Weber number and Murakami and Minemura (1977, 1978) suggest that the ratio of the diameter of the largest bubbles to survive the inlet shear,  $2R_M$ , to the blade spacing,  $h_1$ , will be a function of a Weber number,  $We = \rho \Omega^2 R_{T1}^2 h_1 / S$ . Figure 1 presents some data on  $2R_M/h_1$  taken by Murakami and Minemura for both centrifugal and axial flow pumps.

The size of the bubbles in the blade passages is important because it is the migration and coalesence of these bubbles that appears to cause degradation in the performance. Since the velocity of the relative motion between the bubbles and the liquid is proportional to the bubble size raised to some power which depends on the Reynolds number regime, it follows that the larger the bubbles the more likely it is that large voids will form within the blade passage due to migration of the bubbles toward regions of lower pressure (Furuya 1985, Furuya and Maekawa 1985). As Patel and Runstadler (1978) observed during experiments on centrifugal pumps and rotating passages, regions of low pressure occur not only on the suction sides of the blades but also under the shroud of a centrifugal pump. These large voids can cause substantial changes in the deviation angle of the flow leaving the impeller and hence alter the pump performance in a significant way. This mechanism of head degradation is probably significant not only for gas/liquid flows but also for cavitating flows. In gas/liquid flows the higher the velocity the greater the degree of bubble fission at inlet and the smaller the bubbles. But the force acting on the bubbles is also greater for the higher velocity flows and so the net result is not obvious. One can only conclude that both processes, inlet fission and blade passage migration, may be important and deserve further study along the lines begun by Murakami and Minemura.

At the beginning of this section we discussed the application of the Rayleigh-Plesset equation to study the behavior of individual cavitating bubbles. One area in which such an analysis has been useful is in evaluating the differences in the cavitation occurring in different liquids and in the same liquid at different temperatures. These issues will be addressed in the next section. In the subsequent section we turn our attention to the free streamline methods which have been developed to model the flows which occur when large attached cavities or gas-filled voids occur on the blades of a turbomachine.