Pumps in Series

Pumps are often connected in series in order to produce a larger head rise than any one of the pumps or impellers could achieve alone. The usual arrangement is shown in Figure 1 (left). Sometimes physical limitations, for example on the pump diameter, lead to designs involving many, many stages as in down-hole oil well pumps; shown in Figure 1 (right) is one stage in an oil well pump stack that might involve as 20 or 30 stages.

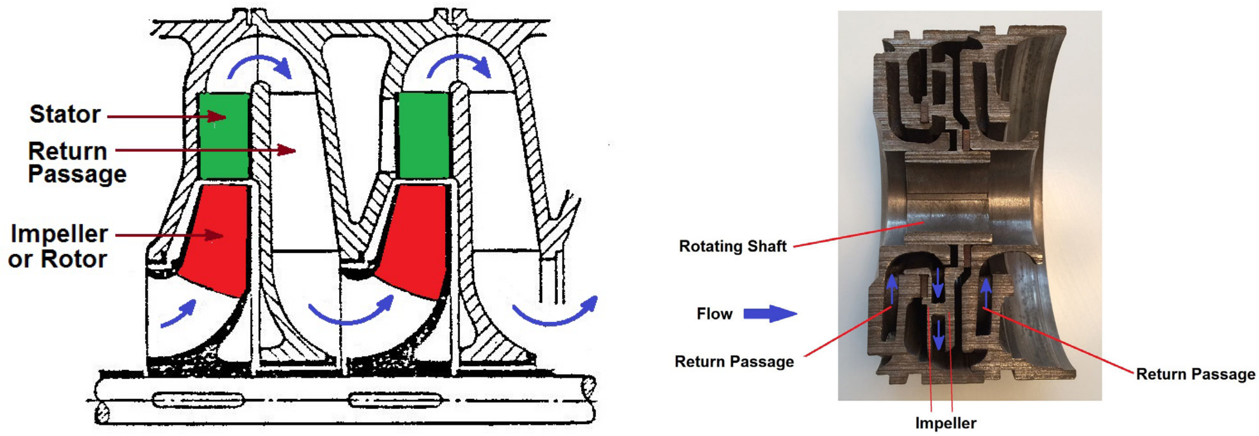

Figure 1: Left: Typical multistage centrifugal pump. Right: One stage of a down-hole oil well pump.

The first step in designing a pump stack is to calculate the overall or total specific speed, N_T, given by

$$N_T = \frac{\Omega Q^{\frac{1}{2}}}{(gH_T)^{\frac{3}{4}}} \tag{Mbbk1}$$

where Ω (in rad/s) is the rotational speed (assumed the same for all impellers in the stack) , Q (in m^3/s) is the flow rate (assumed the same for all impellers in the stack) and H_T (in m) is the total head rise produced by the whole stack. The second step is to decide on the design of an impeller, or more specifically on a design specific speed, N_1, associated with each individual stage in the stack. For example, if each stage is to consist of a centrifugal impeller then, according to section Mbbd, an appropriate choice for N_1 would be about 0.5. On the other hand if each stage is to consist of an axial flow impeller then, according to section Mbbd, an appropriate choice for N_1 would be about 4.0. Having decided on an N_1 it then follows that the number of stages in the stack, n, should be

$$n = \left(\frac{N_1}{N_T}\right)^{\frac{4}{3}} \tag{Mbbk2}$$

since the head rise across each stage will be H_T/n.

Performance of Pumps in Series

Another common configuration is the series operation of two or more pumps with their *own motors*. It is therefore useful to examine some of the features of such operation. For simplicity we examine two pumps operating in series; larger numbers can be treated by a simple extension of the methodology outlined here. The two pumps are assumed to have the same non-dimensional head/flow characteristic, a plot of the reduced head rise, $h = H/N^2$, against the reduced flow rate, $q = Q/N$ where, in these expressions H is the head rise across each pump (in m), Q is the flow rate (in m^3/s), and N is the impeller rotational speed (in rad/s).

First we consider the two pumps, labeled A and B, operating at the same speed, N and, in the absence of cavitation, we denote the identical operating points consisting of a common flow rate Q ($q = Q/N$) and each pump with its own individual total head rise H ($h = H/N^2$) so that the combined total head rise is $2H$. We will assume that the pump characteristic through the operating point can be locally approximated by a straight line and that the negative slope of that straight line is denoted by $S = -\delta h/\delta q = (-\delta H/\delta Q)/N$. Furthermore we will assume that the final discharge proceeds through a pipeline or other device with a quadratic hydraulic resistance, R (total head loss equal to R times the square of the flow rate).

Now consider what happens when the speed of pump B is increased by a small quantity δN to $N + \delta N$ while the speed of pump A remains unchanged. The increase in the total head rise across pump A will be denoted by δH_A and that across pump B by δH_B. The flow rate increase will be denoted by δQ so that the new common flow rate through both pumps will be $Q + \delta Q$. We will proceed to find the relations between δN, δH_A, δH_B and δQ.

The operating points of both pumps are assumed to track along the reduced pump characteristic. In the case of pump A whose speed does not change this means that

$$\frac{H + \delta H_A}{N^2} - \frac{H}{N^2} = S\left\{\frac{Q}{N} - \frac{(Q + \delta Q)}{N}\right\} \tag{Mbbk3}$$

or

$$\delta H_A = -SN\delta Q \tag{Mbbk4}$$

In the case of pump B whose speed does increase by δN it means that

$$\frac{H + \delta H_B}{(N + \delta N)^2} - \frac{H}{N^2} = S\left\{\frac{Q}{N} - \frac{(Q + \delta Q)}{(N + \delta N)}\right\} \tag{Mbbk5}$$

or

$$\delta H_B = \{2H + SNQ\}\frac{\delta N}{N} - SN\delta Q \tag{Mbbk6}$$

where we have neglected all quadratic combinations of the incremental changes. Combining equations (Mbbk4) and (Mbbk6) the combined total head rise of the two pumps is now

$$2H + \delta H_A + \delta H_B = 2H + \{2H + SNQ\}\frac{\delta N}{N} - 2SN\delta Q \tag{Mbbk7}$$

It is, of course, possible to select from a number of alternative boundary conditions at the pump discharge. For present purposes, we choose to apply a quadratic hydraulic resistance, R, such that the head loss downstream of the discharge is equal to R times the square of the flow rate and downstream of that the total head remains constant. Consequently

$$\delta H_A + \delta H_B = \{2H + SNQ\}\frac{\delta N}{N} - 2SN\delta Q = 2RQ\delta Q \tag{Mbbk8}$$

where quadratic combinations of the incremental quantities have again been neglected. It follows that

$$\frac{\delta Q}{Q} = \frac{\left\{1+\frac{2H}{SNQ}\right\}}{\left\{1+\frac{RQ}{SN}\right\}}\frac{\delta N}{2N} \tag{Mbbk9}$$

and substituting back into equations (Mbbk4) and (Mbbk6) the changes in the total head rises become

$$\frac{\delta H_A}{H} = -\frac{\left\{1+\frac{SNQ}{2H}\right\}}{\left\{1+\frac{RQ}{SN}\right\}}\frac{\delta N}{N} \tag{Mbbk10}$$

and

$$\frac{\delta H_B}{H} = \frac{\left\{1+\frac{SNQ}{2H}\right\}\left\{1+\frac{2RQ}{SN}\right\}}{\left\{1+\frac{RQ}{SN}\right\}}\frac{\delta N}{N} \tag{Mbbk11}$$

The results in equations (Mbbk9), (Mbbk10) and (Mbbk11) exhibit several different asymptotic limits that are useful to describe:

- For pumps operating at the high head/low flow end of their characteristic where dH/dQ and S are small (explicitly $|S| \ll H/NQ$) it follows from the above results that if $RQ \ll |S|N$

$$\frac{\delta Q}{Q} \to \frac{H}{SNQ}\frac{\delta N}{N} \quad ; \quad \frac{\delta H_A}{H} \to \frac{\delta N}{N} \quad ; \quad \frac{\delta H_B}{H} \to \frac{\delta N}{N} \tag{Mbbk12}$$

  or if $RQ \gg |S|N$ then

$$\frac{\delta Q}{Q} \to \frac{H}{RQ^2}\frac{\delta N}{N} \quad ; \quad \frac{\delta H_A}{H} \to -\frac{SN}{RQ}\frac{\delta N}{N} \quad ; \quad \frac{\delta H_B}{H} \to 2\frac{\delta N}{N} \tag{Mbbk13}$$

- In the limit of large discharge line resistance (specifically $|R| \gg |S|N/Q$) the system responses reduce to

$$\frac{\delta Q}{Q} \to 0 \quad ; \quad \frac{\delta H_A}{H} \to 0 \quad ; \quad \frac{\delta H_B}{H} \to \left\{2+\frac{SNQ}{H}\right\}\frac{\delta N}{2N} \tag{Mbbk14}$$

- In the other limit of a small discharge line resistance (specifically $|R| \ll |S|N/Q$)

$$\frac{\delta Q}{Q} \to \left\{1+\frac{2H}{SNQ}\right\}\frac{\delta N}{2N} \quad ; \quad \frac{\delta H_A}{H} \to -\left\{1+\frac{SNQ}{2H}\right\}\frac{\delta N}{N} \quad ; \quad \frac{\delta H_B}{H} \to \left\{1+\frac{SNQ}{2H}\right\}\frac{\delta N}{N} \tag{Mbbk15}$$

As a numerical example, consider the case in which $H = 45.6m$, $Q = 0.874m^3/s$, $-(dH/dQ) = 37.4s/m^2$

from which it follows that $H/SNQ = 1.395$. Then, according to the above formula, a 5% increase in the speed of pump B ($\delta N/N = 0.05$) would lead to the following:

- In the limit of a large discharge line resistance:

$$\frac{\delta Q}{Q} \to 0 \quad ; \quad \frac{\delta H_A}{H} \to 0 \quad ; \quad \frac{\delta H_B}{H} \to 2.717\frac{\delta N}{N} = 0.1358 \tag{Mbbk16}$$

  so the fractional increase in both the total head and the head of pump A are disappearing; on the other hand the fractional increase in the head of pump B is almost three times the fractional increase in the speed of pump B.

- In the limit of a small discharge line resistance:

$$\frac{\delta Q}{Q} \rightarrow 3.79\frac{\delta N}{N} = 0.1895 \quad ; \quad \frac{\delta H_A}{H} \rightarrow -1.358\frac{\delta N}{N} = -0.0679 \quad ; \quad \frac{\delta H_B}{H} \rightarrow 1.358\frac{\delta N}{N} = 0.0679 \tag{Mbbk17}$$

  so the fractional increase in the flow is almost four times the fractional increase in speed of pump B while the fractional changes in the pump heads are slightly larger than the fractional increase in the speed of pump B.