Pumps in Parallel

Pumps are often connected in parallel in order to accommodate a large range of flow rates with the same head rise; one example is a waste water treatment station handling flows that may vary by an order of magnitude with annual, monthly or daily demand. While is is possible to efficiently handled flow rates varying by 30-40% by varying the impeller rotation speed, N, it is not possible to handle larger flow rate variations without encountering serious vibration problems or pump failure.

It is therefore useful to examine some of the features of parallel operation. For simplicity we examine two pumps operating in parallel; larger numbers can be treated by a simple extension of the methodology outlined here. The two pumps are assumed to have the same non-dimensional head/flow characteristic, a plot of the reduced head rise, $h = H/N^2$, against the reduced flow rate, $q = Q/N$ where, in these expressions H is the total head rise across a pump (in m), Q is the flow rate (in m^3/s), and N is the rotational speed (in rad/s) of the impellers.

First we consider the two pumps, labeled A and B, operating at the same speed, N and, in the absence of cavitation, we denote the identical operating points consisting of a total head rise H ($h = H/N^2$) and a individual flow rate Q ($q = Q/N$) with a combined flow rate of $2Q$. We will assume that the pump characteristic through that operating point can be locally approximated by a straight line and that the negative slope of that straight line is denoted by $S = -\delta h/\delta q = (-\delta H/\delta Q)/N$. Furthermore we will assume that the combined discharge proceeds through a pipeline or other device with a quadratic hydraulic resistance, R.

Now consider what happens when the speed of pump B is increased by a small quantity δN to $N + \delta N$. The increase in the total head rise across the pumps will be denoted by δH and the flow rate increases that result will be denoted by δQ_A and δQ_B for pumps A and B respectively so that the new total flow rate through both pumps will be $2Q + \delta Q_A + \delta Q_B$. We will proceed to find the relations between δN, δH, δQ_A, and δQ_B.

The operating points of both pumps are assumed to track along the reduced pump characteristic. In the case of pump A whose speed does not change this means that

$$\frac{H + \delta H}{N^2} - \frac{H}{N^2} = S\left\{\frac{Q}{N} - \frac{(Q + \delta Q_A)}{N}\right\} \tag{Mbbh1}$$

or

$$\delta Q_A = -\frac{\delta H}{SN} \tag{Mbbh2}$$

In the case of pump B whose speed does increase by δN it means that

$$\frac{H + \delta H}{(N + \delta N)^2} - \frac{H}{N^2} = S\left\{\frac{Q}{N} - \frac{(Q + \delta Q_B)}{(N + \delta N)}\right\} \tag{Mbbh3}$$

or

$$\delta Q_B = -\frac{\delta H}{SN} + \frac{\delta N}{N}\left\{Q + \frac{2H}{SN}\right\} \tag{Mbbh4}$$

where we have neglected all quadratic combinations of the incremental changes. Combining equations (Mbbh2) and (Mbbh4) the combined flow rate downstream of the pumps is now

$$2Q + \delta Q_A + \delta Q_B = 2Q - \frac{2\delta H}{SN} + \frac{\delta N}{N}\left\{Q + \frac{2H}{SN}\right\} \tag{Mbbh5}$$

While noting that alternative boundary conditions downstream of the discharge line or device are possible and that the discharge line or device is assumed to have a quadratic hydraulic resistance, R (head loss equal to R times the square of the flow rate), it is here assumed that the total head downstream of that device remains unchanged so that

$$\delta H = 4RQ(\delta Q_A + \delta Q_B) = 4RQ \left[\frac{2\delta H}{SN} - \frac{\delta N}{N} \left\{ Q + \frac{2H}{SN} \right\} \right] \tag{Mbbh6}$$

where quadratic combinations of the incremental quantities have again been neglected. It follows that

$$\frac{\delta H}{H} = \frac{\left\{1 + \frac{SNQ}{2H}\right\}}{\left\{1 - \frac{SN}{8RQ}\right\}} \frac{\delta N}{N} \tag{Mbbh7}$$

and substituting back into equations (Mbbh2) and (Mbbh4) the changes in the two flow rates become

$$\frac{\delta Q_A}{Q} = -\frac{\left\{1 + \frac{2H}{SNQ}\right\}}{\left\{1 - \frac{SN}{8RQ}\right\}} \frac{\delta N}{2N} \tag{Mbbh8}$$

and

$$\frac{\delta Q_B}{Q} = \frac{\left\{1 + \frac{2H}{SNQ}\right\}\left\{1 - \frac{SN}{4RQ}\right\}}{\left\{1 - \frac{SN}{8RQ}\right\}} \frac{\delta N}{2N} \tag{Mbbh9}$$

The results in equations (Mbbh7), (Mbbh8) and (Mbbh9) exhibit several different asymptotic limits that are useful to describe:

- For pumps operating at the high head/low flow end of their characteristic where dH/dQ and S are small (explicitly $|S| \ll H/NQ$ but with $|R| > |S|N/Q$) it follows from the above results that

$$\frac{\delta H}{H} \rightarrow \frac{\delta N}{N} \quad ; \quad \frac{\delta Q_A}{Q} \rightarrow -\frac{H}{SNQ} \frac{\delta N}{N} \quad ; \quad \frac{\delta Q_B}{Q} \rightarrow \frac{H}{SNQ} \frac{\delta N}{N} \tag{Mbbh10}$$

  Consequently, under these conditions, a small change in the speed of one of the pumps leads to large fractional changes in the flow rates through the pumps with the changes being of opposite sign in the two pumps. This suggests a violent flow rate oscillation with flow oscillating from one pump to the other.

- In the limit of large discharge line resistance (specifically $|R| \gg |S|N/Q$) the system responses reduce to

$$\frac{\delta H}{H} \rightarrow \left\{1 + \frac{SNQ}{2H}\right\} \frac{\delta N}{N} \quad ; \quad \frac{\delta Q_A}{Q} \rightarrow -\left\{1 + \frac{2H}{SNQ}\right\} \frac{\delta N}{2N} \quad ; \quad \frac{\delta Q_B}{Q} \rightarrow \left\{1 + \frac{2H}{SNQ}\right\} \frac{\delta N}{2N} \tag{Mbbh11}$$

  Under these conditions the fractional change in the head is less than the fractional change in the speed (since S is normally negative) while the incremental changes in the flow rates are equal but of opposite sign. Again this suggests a susceptibility to oscillation in the flow from one pump to the other.

- In the other limit of a small discharge line resistance (specifically $|R| \ll |S|N/Q$)

$$\frac{\delta H}{H} \rightarrow 0 \quad ; \quad \frac{\delta Q_A}{Q} \rightarrow 0 \quad ; \quad \frac{\delta Q_B}{Q} \rightarrow \left\{1 + \frac{2H}{SNQ}\right\} \frac{\delta N}{N} \tag{Mbbh12}$$

  and the magnitude of the fractional change in the flow rate of pump B relative to the fractional change in the speed of that pump will depend on the magnitude of $H/|S|NQ$.

As a numerical example, consider the case in which $H = 45.6m$, $Q = 0.874m^3/s$, $-(dH/dQ) = 37.4s/m^2$ from which it follows that $H/SNQ = 1.395$. Then, according to the above formula, a 5% increase in the speed of pump B ($\delta N/N = 0.05$) would lead to the following:

- In the limit of a large discharge line resistance:

$$\frac{\delta H}{H} \rightarrow 1.358\frac{\delta N}{N} = 0.0679 \quad ; \quad \frac{\delta Q_A}{Q} \rightarrow -1.895\frac{\delta N}{N} = -0.0947 \quad ; \quad \frac{\delta Q_B}{Q} \rightarrow 1.895\frac{\delta N}{N} = 0.0947 \tag{Mbbh13}$$

  so the fractional increase in the total head is somewhat larger than the fractional increase in the speed of pump B; on the other hand the fractional changes in the flow rates through the two pumps is almost twice the fractional increase in the speed of pump B.

- In the limit of a small discharge line resistance:

$$\frac{\delta H}{H} \rightarrow 0 \quad ; \quad \frac{\delta Q_A}{Q} \rightarrow 0 \quad ; \quad \frac{\delta Q_B}{Q} \rightarrow 3.79\frac{\delta N}{N} = 0.1895 \tag{Mbbh14}$$

  so the fractional increases in the total head and in the flow through pump A become negligible while the fractional increase in flow rate through pump B is almost 4 times the fractional increase in the speed of pump B.