Geometric Notation

The geometry of a generalized turbomachine rotor is sketched in figure 1, and consists of a set of rotor blades (number = Z_R) attached to a hub and operating within a static casing. The radii of the inlet blade tip, inlet blade hub, discharge blade tip, and discharge blade hub are denoted by R_{T1}, R_{H1}, R_{T2}, and R_{H2}, respectively. The discharge blade passage is inclined to the axis of rotation at an angle, ϑ, which would be close to 90° in the case of a centrifugal pump, and much smaller in the case of an axial flow machine. In practice, many pumps and turbines are of the "mixed flow" type , in which the typical or mean discharge flow is at some intermediate angle, $0 < \vartheta < 90°$.

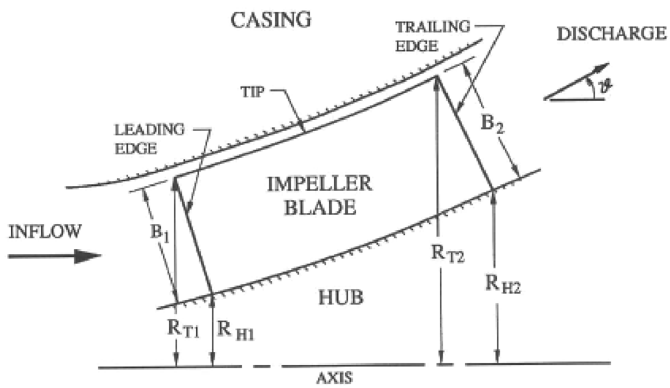

Figure 1: Cross-sectional view through the axis of a pump impeller.

The flow through a general rotor is normally visualized by developing a meridional surface (figure 2), that can either correspond to an axisymmetric stream surface, or be some estimate thereof. On this meridional surface (see figure 2) the fluid velocity in a non-rotating coordinate system is denoted by $v(r)$ (with subscripts 1 and 2 denoting particular values at inlet and discharge) and the corresponding velocity relative to the rotating blades is denoted by $w(r)$. The velocities, v and w, have components v_θ and w_θ in the circumferential direction, and v_m and w_m in the meridional direction. Axial and radial components are denoted by the subscripts a and r. The velocity of the blades is Ωr. As shown in figure 2, the flow angle $\beta(r)$ is defined as the angle between the relative velocity vector in the meridional plane and a plane perpendicular to the axis of rotation. The blade angle $\beta_b(r)$ is defined as the inclination of the tangent to the blade in the meridional plane and the plane perpendicular to the axis of rotation. If the flow is precisely parallel to the blades, $\beta = \beta_b$. Specific values of the blade angle at the leading and trailing edges (1 and 2) and at the hub and tip (H and T) are denoted by the corresponding suffices, so that, for example, β_{bT2} is the blade angle at the discharge tip.

At the leading edge it is important to know the angle $\alpha(r)$ with which the flow meets the blades, and, as defined in figure 3,

$$\alpha(r) = \beta_{b1}(r) - \beta_1(r). \quad \text{(Mbba1)}$$

This angle, α, is called the incidence angle, and, for simplicity, we shall denote the values of the incidence angle at the tip, $\alpha(R_{T1})$, and at the hub, $\alpha(R_{H1})$, by α_T and α_H, respectively. Since the inlet flow can often be assumed to be purely axial ($v_1(r) = v_{a1}$ and parallel with the axis of rotation), it follows that $\beta_1(r) = \tan^{-1}(v_{a1}/\Omega r)$, and this can be used in conjunction with equation (Mbba1) in evaluating the incidence angle for a given flow rate.

The incidence angle should not be confused with the "angle of attack", which is the angle between the incoming relative flow direction and the chord line (the line joining the leading edge to the trailing edge).

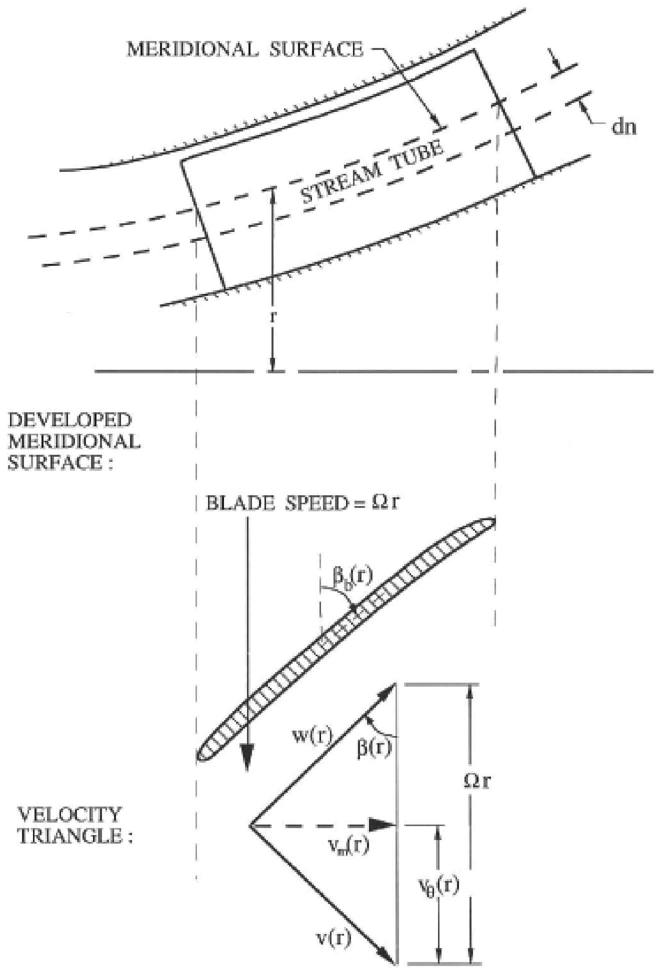

Figure 2: Developed meridional surface and velocity triangle.

Note, however, that, in an axial flow pump with straight helicoidal blades, the angle of attack is equal to the incidence angle.

At the trailing edge, the difference between the flow angle and the blade angle is again important. To a first approximation one often assumes that the flow is parallel to the blades, so that $\beta_2(r) = \beta_{b2}(r)$. A departure from this idealistic assumption is denoted by the deviation angle, $\delta(r)$, where, as shown in figure 3:

$$\delta(r) = \beta_{b2}(r) - \beta_2(r) \qquad \text{(Mbba2)}$$

This is normally a function of the ratio of the width of the passage between the blades to the length of the same passage, a geometric parameter known as the solidity which is defined more precisely below. Other angles, that are often used, are the angle through which the flow is turned, known as the *deflection angle*, $\beta_2 - \beta_1$, and the corresponding angle through which the blades have turned, known as the *camber angle* and denoted by $\theta_c = \beta_{b2} - \beta_{b1}$.

Deviation angles in radial machines are traditionally represented by the *slip velocity*, $v_{\theta s}$, which is the difference between the actual and ideal circumferential velocities of the discharge flow, as shown in figure 4. It follows that

$$v_{\theta s} = \Omega R_2 - v_{\theta 2} - v_{r2} \cot \beta_{b2} \qquad \text{(Mbba3)}$$

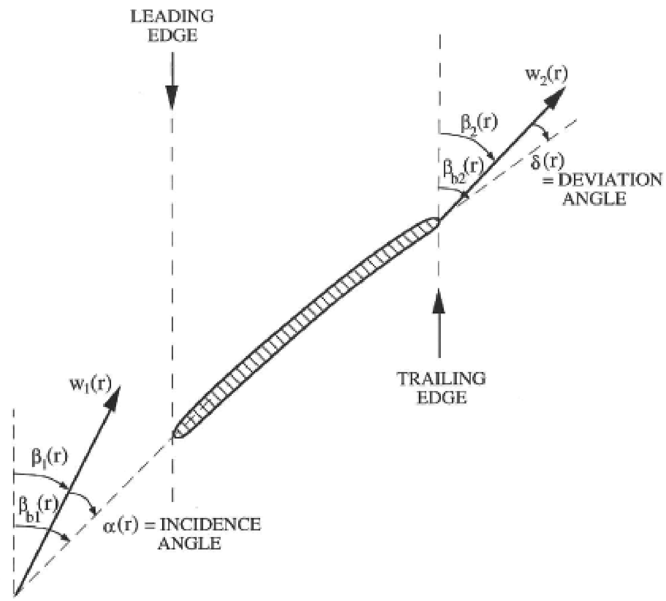

Figure 3: Repeat of figure 2 showing the definitions of the incidence angle at the leading edge and the deviation angle at the trailing edge.

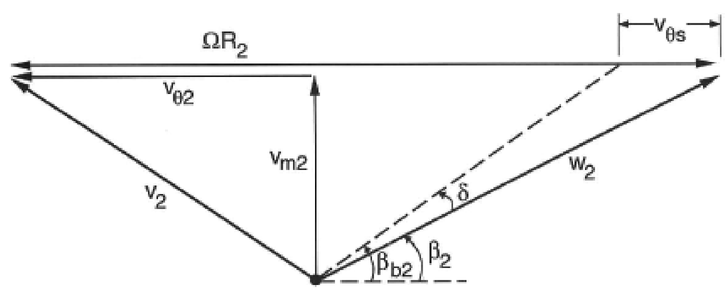

Figure 4: Velocity vectors at discharge indicating the slip velocity, $v_{\theta s}$.

This, in turn, is used to define a parameter known as the *slip factor*, Sf, where

$$Sf = 1 - \frac{v_{\theta s}}{\Omega R_2} = 1 - \phi_2 \left(\cot \beta_2 - \cot \beta_{b2}\right) \qquad \text{(Mbba4)}$$

Other, slightly different "slip factors" have also been used in the literature; for example, Stodola (1927), who originated the concept, defined the slip factor as $1 - v_{\theta s}/\Omega R_2(1 - \phi_2 \cot \beta_{b2})$. However, the definition (Mbba4) is now widely used. It follows that the deviation angle, δ, and the slip factor, Sf, are related by

$$\delta = \beta_{b2} - \cot^{-1}\left(\cot \beta_{b2} + \frac{(1 - Sf)}{\phi_2}\right) \qquad \text{(Mbba5)}$$

where the flow coefficient, ϕ_2, is defined later in section (Mbbc).