Efficiency of Propellers

As described in section (Ddc), the propeller efficiency, η, is the fraction of the shaft energy that is used to propel the vehicle. Consider the simplest model of the propulsion of a boat using a propeller. Figure 1 is a diagram depicting the motion of a propeller blade in which we are now viewing the boat from vertically overhead. The velocity of the blade relative to the hull is denoted by V and this velocity is perpendicular to the direction of motion of the boat, U. The movement of the screw blade produces the lift, L, which propels the boat at its velocity, U, and is equal to the drag on the boat. Consequently is given by

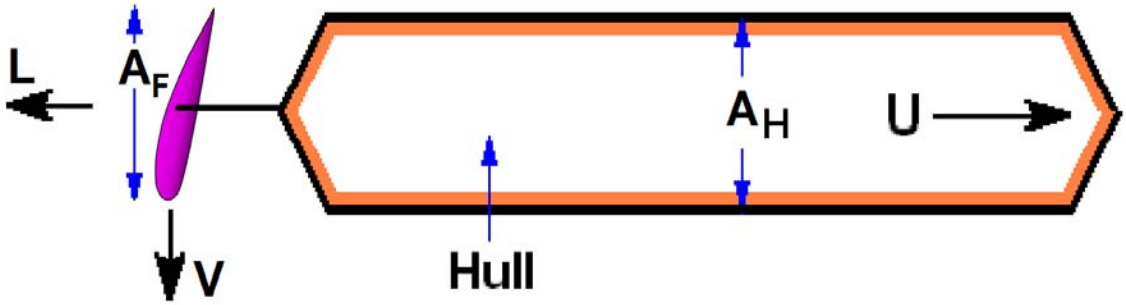

Figure 1: Schematic of a boat propelled by a screw propeller.

$$\frac{1}{2}\rho\, C_{LP} A_P V^2 \;=\; \frac{1}{2}\rho\, C_{DH} A_H U^2 \tag{Mfd1}$$

where C_{LP} is the lift coefficient produced by the screw blade and A_P is the effective planform area of all the screw blades. Hence the velocity ratio V/U is given by

$$\frac{V}{U} \;=\; \left\{ \frac{C_{DH} A_H}{C_{LP} A_P} \right\}^{\frac{1}{2}} \tag{Mfd2}$$

It is important to note that a well-chosen foil shape and angle of attack for the blade, will lead to a value of C_{LP} much larger than C_{DP}.

The rate of work done to move the screw blades through the water is

$$\frac{1}{2}\rho\, C_{DP} A_P V^3 \tag{Mfd3}$$

where C_{DP} is the drag coefficient for the screw blade. Therefore, the propulsion efficiency is given by

$$\eta \;=\; \frac{\frac{1}{2}\rho\, C_{DH} A_H U^3}{\frac{1}{2}\rho\, C_{DH} A_H U^3 + \frac{1}{2}\rho\, C_{DP} A_P V^3} \;=\; \frac{1}{1 \;+\; \frac{V^3}{U^3}\frac{C_{DP} A_P}{C_{DH} A_H}} \tag{Mfd4}$$

and substituting for V/U this becomes

$$\eta \;=\; \frac{1}{1 \;+\; \left\{ \frac{C_{DH} A_H}{C_{LP} A_P} \right\}^{\frac{1}{2}} \frac{C_{DP}}{C_{LP}}} \tag{Mfd5}$$

It is important to note the presence of the drag to lift ratio, C_{DP}/C_{LP}, in the denominator of equation (Mfd5). A well-designed screw blade profile and angle of attack can produce a drag to lift ratio much less

than unity which therefore will produce a much greater propulsion efficiency. This is another demonstartion that a device which uses lift for propulsion (such as a propeller) has the potential for much greater efficiency than one that uses drag for propulsion (such as a paddle). This same property can be seen in many animal propulsion mechanisms and will be detailed in an earlier section.