Three-Dimensional Flow Effects

In this and the sections which follow we briefly survey some of the other important features of the flows through turbomachines. We begin with the three-dimensional characteristics of flows, and a discussion of some of the difficulties encountered in adapting the two-dimensional cascade analyses of the preceding sections to the complex geometry of most turbomachines.

The preceding sections included a description of some of the characteristics of two-dimensional cascade flows in both the axial and radial geometries. It was assumed that the flow in the meridional plane was essentially two-dimensional, and that the effects of the velocities (and the gradients in the velocity or pressure) normal to the meridional surface were neglible. Moreover, it was tacitly assumed that the flow in a real turbomachine could be synthesized using a series of these two-dimensional solutions for each meridional annulus. In doing so it is implicitly assumed that each annulus corresponds to a streamtube such as depicted in figure 1 and that the geometric relations between the inlet location, r_1, and thickness, dr_1, and the discharge thickness, dn, and location, r_2, are known *a priori*. In practice this is not the case and quasi-three-dimensional methods have been developed in order to determine the geometrical relation, $r_2(r_1)$. These methods continue to assume that the streamsurfaces are axisymmetric, and, therefore, neglect the more complicated three-dimensional aspects of the flow exemplified by the secondary flows discussed in section (Mbdf). Nevertheless, these methods allow the calculation of useful turbomachine performance characteristics, particularly under circumstances in which the complex secondary flows are of less importance, such as close to the design condition. When the turbomachine is operating far from the design condition, the flow within a blade passage may have streamsurfaces that are far from axisymmetric.

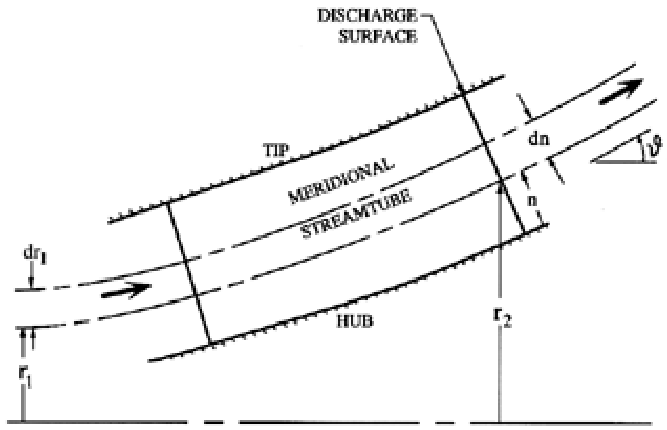

Figure 1: Geometry of a meridional streamtube in a pump impeller.

In the context of axial flow machines, several approximate methods have been employed in order to determine $r_2(r_1)$ as a part of a quasi-three-dimensional solution to the flow. Most of these are based on some application of the condition of radial equilibrium. In its simplest form, the radial equilibrium condition assumes that all of the terms in the equation of motion normal to the axisymmetric streamsurface are negligible, except for the pressure gradient and the centrifugal acceleration terms, so that

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{v_\theta^2}{r} \qquad \text{(Mbdb1)}$$

(The equivalent assumption in a radial machine would be that the axial pressure gradient is zero.) This assumption is differently embedded in several approaches to the solution of the flow. All of these use a condition like equation (Mbdb1) (or some more accurate version) to relate the pressures in the different streamtubes upstream of the rotor (or stator), and a similar condition to connect the pressures in the streamtubes downstream of the rotor (or stator). When these relations are combined with the normal continuity and energy equations for each streamtube (that connect the conditions upstream with those at the downstream location), a complete set of equations is generated, and a solution to the flow can be obtained. In this class of meridional streamtube methods, the velocities normal to meridional streamsurfaces are largely neglected, but the cross-sectional areas of the streamtubes are adjusted to satisfy a condition based on the equation of motion normal to the meridional surface. Notable examples of this class of quasi-three-dimensional solutions are those devised at NASA Lewis by Katsanis and his co-workers (see Stockman and Kramer 1963, Katsanis 1964, Katsanis and McNally 1977).

The following example will illustrate one use of the "radial equilibrium" condition. We shall assume that the inlet flow is in radial equilibrium. This inlet flow is then divided into axisymmetric streamtubes, each with a specific radial location, r_1. Some initial estimate is made of the radial location of each of the streamtubes at discharge (in other words an estimate of the function $r_2(r_1)$). Then an iterative numerical method is employed, in which the total pressure rise through each streamtube is evaluated. Hence, the pressure distribution at discharge can be obtained. Then the width of each tube at discharge is adjusted ($r_2(r_1)$ is adjusted) in order to obtain the required radial pressure gradient between each pair of adjacent streamtubes. Subsequently, the process is repeated until a converged solution is reached. In some simple cases, analytical rather than numerical results can be obtained; an example is given in the next section.

More generally, it should be noted that quasi-three-dimensional analyses of this kind are often used for the design of axial turbomachines. A common objective is to achieve a design in which the total pressure is increasing (or decreasing) with axial position at the same rate at all radii, and, therefore, should be invariant with radial position. Combining this with the condition for radial equilibrium, leads to

$$\frac{d}{dr}\left(v_m^2\right) + \frac{1}{r^2}\frac{d}{dr}\left(r^2 v_\theta^2\right) = 0 \qquad \text{(Mbdb2)}$$

If, in addition, we stipulate that the axial velocity, v_m, must be constant with radius, then equation (Mbdb2) implies that the circumferential velocity, v_θ, must vary like $1/r$. Such an objective is termed a "free vortex" design. Another basic approach is the "forced vortex" design in which the circumferential velocity, v_θ, is proportional to the radius, r; then, according to the above equations, the axial velocity must decrease with r. More general designs in which $v_\theta = ar + b/r$ (a and b being constants) are utilized in practice for the design of axial compressors and turbines, with the objective of producing relatively uniform head rise and velocity at different radii (Horlock 1973). However, in the context of pumps, most of the designs are of the "forced vortex" type; Stepanoff (1948) lists a number of reasons for this historical development. Note that a forced vortex design with a uniform axial velocity would imply helical blades satisfying equation (Mbdb1); thus many pumps have radial distributions of blade angle close to the form of that equation.

Radial equilibrium of the discharge flow may be an accurate assumption in some machines but not in others. When the blade passage is narrow (in both directions) relative to its length, the flow has adequate opportunity to adjust within the impeller or rotor passage, and the condition of radial equilibrium at discharge is usually reasonable. This is approximately the case in all pumps except propeller pumps of low solidity. However, in many compressors and turbines, the blade height is large compared with the chord and a radial equilibrium assumption at discharge is not appropriate. Under these circumstances, a very different approach utilizing an "actuator disc" has been successfully employed. The flows far upstream and downstream of the blade row are assumed to be in radial equilibrium, and the focus is on the adjustment of the flow between these locations and the blade row (see figure 2). The flow through the blade row itself is assumed to be so short that the streamsurfaces emerge at the same radial locations at which they entered;

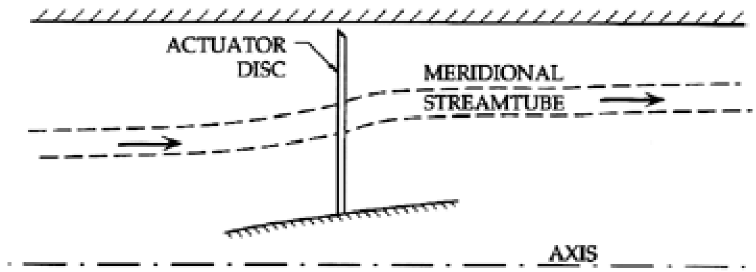

Figure 2: Actuator disc model of an axial blade row with a generic meridional streamtube.

thus the blade row is modeled by an infinitesmally thin "actuator disc". In some respects, the actuator disc approach is the opposite of the radial equilibrium method; in the former, all the streamline adjustment is assumed to occur external to the blade passages whereas, in many radial equilibrium applications, the adjustment all occurs internally.

Since actuator disc methods are rarely applied in the context of pumps we shall not extend the discussion of them further. More detail can be found in texts such as Horlock (1973). We shall, however, provide an example of a radial equilibrium analysis since the results will prove useful in a later chapter.