Cavitation Bubble Dynamics

Two fundamental models for cavitation have been extensively used in the literature. One of these is the spherical bubble model which is most relevant to those forms of bubble cavitation in which nuclei grow to visible, macroscopic size when they encounter a region of low pressure, and collapse when they are convected into a region of higher pressure. For present purposes, we give only the briefest outline of these methods, while referring the reader to the extensive literature for more detail (see, for example, Knapp, Daily and Hammitt 1970, Plesset and Prosperetti 1977, Brennen 1994). The second fundamental methodology is that of free streamline theory, which is most pertinent to flows consisting of attached cavities or vapor-filled wakes; a brief review of this methodology is given in section (Mbes).

Virtually all of the spherical bubble models are based on some version of the Rayleigh-Plesset equation (Plesset and Prosperetti 1977) that defines the relation between the radius of a spherical bubble, $R(t)$, and the pressure, $p(t)$, far from the bubble. In an otherwise quiescent incompressible Newtonian liquid, this equation takes the form

$$\frac{p_B(t) - p(t)}{\rho_L} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu}{R}\frac{dR}{dt} + \frac{2\mathcal{S}}{\rho_L R} \tag{Mbew1}$$

where ν, $\mathcal{S}$, and ρ_L are respectively the kinematic viscosity, surface tension, and density of the liquid. This equation (without the viscous and surface tension terms) was first derived by Rayleigh (1917) and was first applied to the problem of a traveling cavitation bubble by Plesset (1949).

The pressure far from the bubble, $p(t)$, is an input function that could be obtained from a determination of the pressure history that a nucleus would experience as it travels along a streamline. The pressure, $p_B(t)$, is the pressure inside the bubble. It is often assumed that the bubble contains both vapor and noncondensable gas, so that

$$p_B(t) = p_V(T_B) + \frac{3m_G K_G T_B}{4\pi R^3} = p_V(T_\infty) - \rho_L\Theta + \frac{3m_G K_G T_B}{4\pi R^3} \tag{Mbew2}$$

where T_B is the temperature inside the bubble, $p_V(T_B)$ is the vapor pressure, m_G is the mass of gas in the bubble, and K_G is the gas constant. However, it is convenient to use the ambient liquid temperature far from the bubble, T_∞, to evaluate p_V. When this is done, it is necessary to introduce the term, Θ, into equation (Mbew1) in order to correct for the difference between $p_V(T_B)$ and $p_V(T_\infty)$. It is this term, Θ, that is the origin of the thermal effect in cavitation. Using the Clausius-Clapeyron relation,

$$\Theta \cong \frac{\rho_V \mathcal{L}}{\rho_L T_\infty}\left(T_\infty - T_B(t)\right) \tag{Mbew3}$$

where ρ_V is the vapor density and $\mathcal{L}$ is the latent heat.

Note that in using equation (Mbew2) for $p_B(t)$, we have introduced the additional unknown function, $T_B(t)$, into the Rayleigh-Plesset equation (Mbew1). In order to determine this function, it is necessary to construct and solve a heat diffusion equation, and an equation for the balance of heat in the bubble. Approximate solutions to these equations can be written in the following simple form. If the heat conducted into the bubble is equated to the rate of use of latent heat at the interface, then

$$\left(\frac{\partial T}{\partial r}\right)_{r=R} = \frac{\rho_V \mathcal{L}}{k_L}\frac{dR}{dt} \tag{Mbew4}$$

where $(\partial T/\partial r)_{r=R}$ is the temperature gradient in the liquid at the interface and k_L is the thermal conductivity of the liquid. Moreover, an approximate solution to the thermal diffusion equation in the liquid

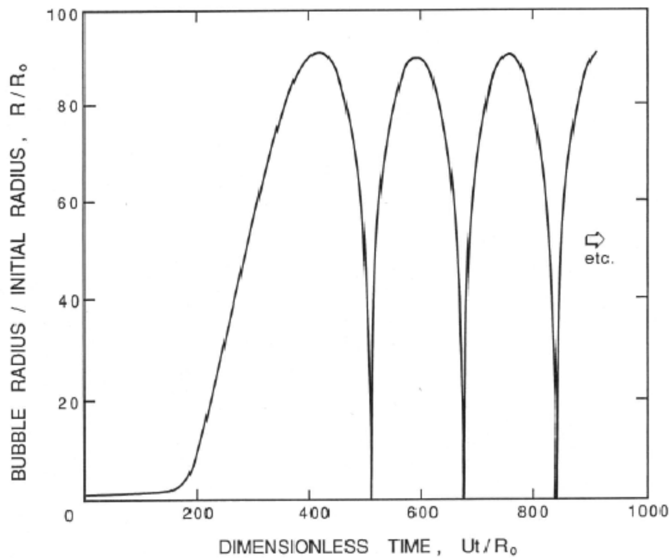

Figure 1: Typical solution, $R(t)$, of the Rayleigh-Plesset equation for a spherical bubble originating from a nucleus of radius, R_0. The nucleus enters a low pressure region at a dimensionless time of 0 and is convected back to the original pressure at a dimensionless time of 500. The low pressure region is sinusoidal and symmetric about a dimensionless time of 250.

is

$$\left(\frac{\partial T}{\partial r}\right)_{r=R} = \frac{(T_\infty - T_B(t))}{(\alpha_L t)^{\frac{1}{2}}} \tag{Mbew5}$$

where α_L is the thermal diffusivity of the liquid ($\alpha_L = k_L/\rho_L c_{PL}$ where c_{PL} is the specific heat of the liquid) and t is the time from the beginning of bubble growth or collapse. Using equations (Mbew4) and (Mbew5) in equation (Mbew3), the thermal term can be approximated as

$$\Theta = \Sigma(T_\infty) t^{\frac{1}{2}} \frac{dR}{dt} \tag{Mbew6}$$

where

$$\Sigma(T_\infty) = \frac{\rho_V^2 \mathcal{L}^2}{\rho_L^2 c_{PL} T_\infty \alpha_L^{\frac{1}{2}}} \tag{Mbew7}$$

In section (Mber), we shall utilize these relations to evaluate the thermal suppression effects in cavitating pumps.

For present purposes, it is useful to illustrate some of the characteristic features of solutions to the Rayleigh-Plesset equation in the absence of thermal effects ($\Theta = 0$ and $T_B(t) = T_\infty$). A typical solution of $R(t)$ for a nucleus convected through a low pressure region is shown in figure 1. Note that the response of the bubble is quite nonlinear; the growth phase is entirely different in character from the collapse phase. The growth is steady and controlled; it rapidly reaches an asymptotic growth rate in which the dominant terms of the Rayleigh-Plesset equation are the pressure difference, $p_V - p$, and the second term on the right-hand side so that

$$\frac{dR}{dt} \Rightarrow \left[\frac{2(p_V - p)}{3\rho_L}\right]^{\frac{1}{2}} \tag{Mbew8}$$

Note that this requires the local pressure to be less than the vapor pressure. For traveling bubble cavitation, the typical tension ($p_V - p$) will be given nondimensionally by ($-C_{pmin} - \sigma$) (see equations (Mbeb2) and (Mbeb4) so the typical growth rate is given by

$$\frac{dR}{dt} \propto (-C_{pmin} - \sigma)^{\frac{1}{2}} U \tag{Mbew9}$$

While this growth rate may appear, superficially, to represent a relatively gentle process, it should be recognized that it corresponds to a volume that is increasing like t^3. Cavitation growth is therefore an explosive process to be contrasted with the kind of boiling growth that occurs in a kettle on the stove in which dR/dt typically behaves like $t^{-\frac{1}{2}}$. The latter is an example of the kind of thermally inhibited growth discussed in section (Mber).

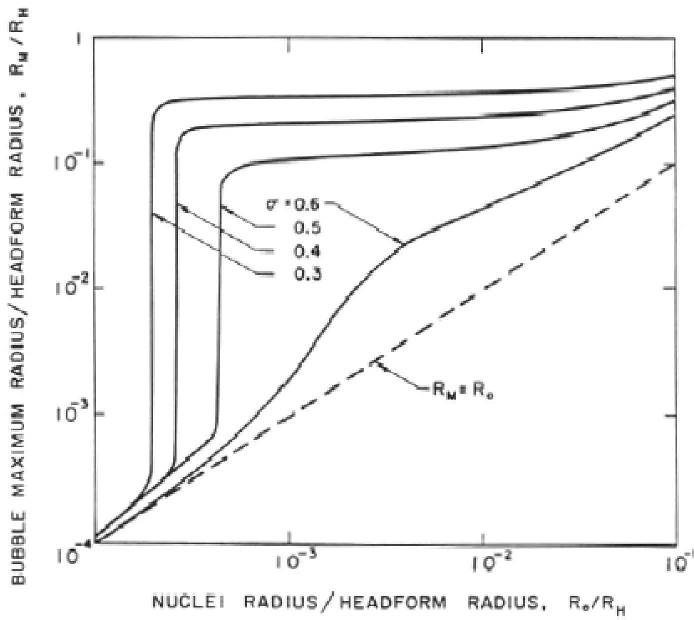

Figure 2: The maximum size to which a cavitation bubble grows (according to the Rayleigh-Plesset equation), R_M, as function of the original nuclei size, R_0, and the cavitation number, σ, in the flow around an axisymmetric headform of radius, R_H, with $\mathcal{S}/\rho_L R_H U^2 = 0.000036$ (from Ceccio and Brennen 1991).

It follows that we can estimate the typical maximum size of a cavitation bubble, R_M, given the above growth rate and the time available for growth. Numerical calculations using the full Rayleigh-Plesset equation show that the appropriate time for growth is the time for which the bubble experiences a pressure below the vapor pressure. In traveling bubble cavitation we may estimate this by knowing the shape of the pressure distribution near the minimum pressure point. We shall represent this shape by

$$C_p = C_{pmin} + C_{p*}(s/D)^2 \qquad \text{(Mbew10)}$$

where s is a coordinate measured along the surface, D is the typical dimension of the body or flow, and C_{p*} is some known constant of order one. Then, the time available for growth, t_G, is given approximately by

$$t_G \approx \frac{2D\,(\sigma - C_{pmin})^{\frac{1}{2}}}{C_{p*}^{\frac{1}{2}} U\,(1 + C_{pmin})^{\frac{1}{2}}} \qquad \text{(Mbew11)}$$

and therefore

$$\frac{R_M}{D} \approx \frac{2\,(-\sigma - C_{pmin})}{C_{p*}^{\frac{1}{2}}\,(1 + C_{pmin})^{\frac{1}{2}}} \qquad \text{(Mbew12)}$$

Note that this is independent of the size of the original nucleus.

One other feature of the growth process is important to mention. It transpires that because of the stabilizing influence of the surface tension term, a particular tension, $(p_V - p)$, will cause only bubbles larger than a certain critical size to grow explosively (Blake 1949). This means that, for a given cavitation number, only nuclei larger than a certain critical size will achieve the growth rate necessary to become macroscopic cavitation bubbles. A decrease in the cavitation number will activate smaller nuclei, thus increasing the volume of cavitation. This phenomenon is illustrated in figure 2 which shows the maximum size of a cavitation bubble, R_M, as a function of the size of the original nucleus and the cavitation number for a typical flow around an axisymmetric headform. The vertical parts of the curves on the left of the figure represent the values of the critical nuclei size, R_C, that are, incidentally, given simply by the expression

$$R_C \approx \kappa \mathcal{S}/\rho_L U^2 \left(-\sigma - C_{pmin}\right) \qquad \text{(Mbew13)}$$

where the factor κ is roughly unity (Ceccio and Brennen 1991). Note also from figure 2 that all the unstable nuclei grow to roughly the same size as anticipated earlier.

Turning now to the collapse, it is readily seen from figure 1 that cavitation bubble collapse is a catastrophic phenomenon in which the bubble, still assumed spherical, reaches a size very much smaller than the original nucleus. Very high accelerations and pressures are generated when the bubble becomes very small. However, if the bubble contains any noncondensable gas at all, this will cause a rebound as shown in figure 1. Theoretically, the spherical bubble will undergo many cycles of collapse and rebound. In practice, a collapsing bubble becomes unstable to nonspherical disturbances, and essentially shatters into many smaller bubbles in the first collapse and rebound. The resulting cloud of smaller bubbles rapidly disperses. Whatever the deviations from the spherical shape, the fact remains that the collapse is a violent process that produces noise and the potential for material damage to nearby surfaces. We proceed to examine both of these consequences in the two sections which follow.