Cascades

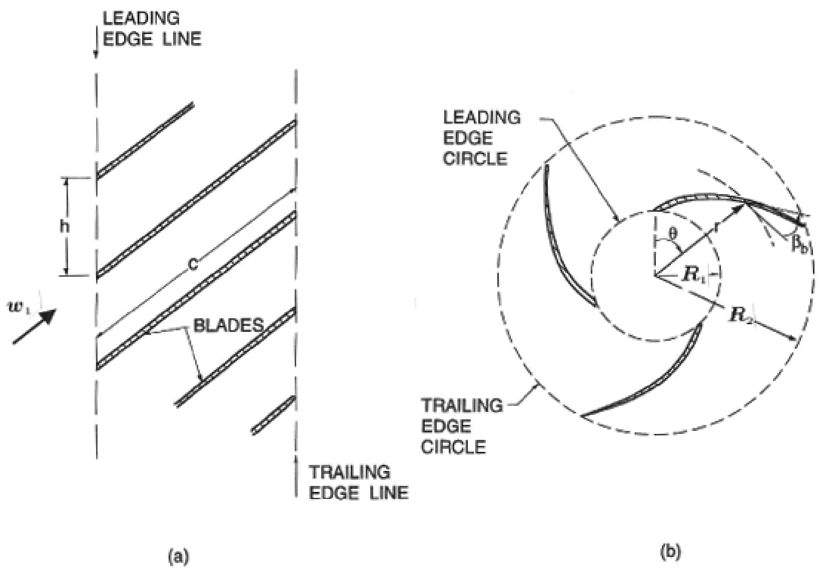

Figure 1: Schematics of (a) a linear cascade and (b) a radial cascade.

We now turn to some specific geometric features that occur frequently in discussions of pumps and other turbomachines. In a purely axial flow machine, the development of a cylindrical surface within the machine produces a *linear cascade* of the type shown in figure 1(a). The centerplane of the blades can be created using a "generator", say $z = z^*(r)$, which is a line in the $rz-$plane. If this line is rotated through a helical path, it describes a helicoidal surface of the form

$$z = z^*(r) + \frac{h_p\theta}{2\pi} \quad \text{(Mbbb1)}$$

where h_p is the "pitch" of the helix. Of course, in many machines, the pitch is also a function of θ so that the flow is turned by the blades. If, however, the pitch is constant, the development of a cylindrical surface will yield a cascade with straight blades and constant blade angle, β_b. Moreover, the blade thickness is often neglected, and the blades in figure 1(a) then become infinitely thin lines. Such a cascade of infinitely thin, flat blades is referred to as a *flat plate cascade*.

It is convenient to use the term "simple" cascade to refer to those geometries for which the blade angle, β_b, is constant whether in an axial, radial, or mixed flow machine. Clearly, the flat plate cascade is the axial flow version of a simple cascade.

Now compare the geometries of the cascades at different radii within an axial flow machine. Later, we analyse the cavitating flow occurring at different radii. Often the pitch at a given axial position is the same at all radii. Then it follows that the radial variation in the blade angle, $\beta_b(r)$, must be given by

$$\beta_b(r) = \tan^{-1}\left[\frac{R_T \tan\beta_{bT}}{r}\right] \quad \text{(Mbbb2)}$$

where β_{bT} is the blade angle at the tip, $r = R_T$.

In a centrifugal machine in which the flow is purely radial, a cross-section of the flow would be as shown in figure 1(b), an array known as a *radial cascade*. In a *simple* radial cascade, the angle, β_b, is uniform along the length of the blades. The resulting blade geometry is known as a logarithmic spiral, since it follows that the coordinates of the blades are given by the equation

$$\theta - \theta_0 = A \ln r \tag{Mbbb3}$$

where $A = \cot \beta_b$ and θ_0 are constants. Logarithmic spiral blades are therefore equivalent to straight blades in a linear cascade. Note that a fluid particle in a flow of uniform circulation and constant source strength at the origin will follow a logarithmic spiral since all velocities will be of the form C/r where C is a uniform constant.

In any of type of pump, the ratio of the length of a blade passage to its width is important in determining the degree to which the flow is guided by the blades. The solidity, s, is the geometric parameter that is used as a measure of this geometric characteristic, and s can be defined for any *simple* cascade as follows. If we identify the difference between the θ coordinates for the same point on adjacent blades (call this $\Delta\theta_A$) and the difference between the θ coordinates for the leading and trailing edges of a blade (call this $\Delta\theta_B$), then the solidity for a simple cascade is defined by

$$s = \frac{\Delta\theta_B}{\Delta\theta_A \cos \beta_b} \tag{Mbbb4}$$

Applying this to the linear cascade of figure 1(a), we find the familiar

$$s = c/h \tag{Mbbb5}$$

In an axial flow pump this corresponds to $s = Z_R c/2\pi R_{T1}$, where c is the chord of the blade measured in the developed meridional plane of the blade tips. On the other hand, for the radial cascade of figure 1(b), equation (Mbbb4) yields the following expression for the solidity:

$$s = Z_R \ell n \left(R_2/R_1\right) / 2\pi \sin \beta_b \tag{Mbbb6}$$

which is, therefore, geometrically equivalent to c/h in the linear cascade.

In practice, there exist many "mixed flow" pumps whose geometries lie between that of an axial flow machine ($\vartheta = 0$, figure 1, section (Mbba)) and that of a radial machine ($\vartheta = \pi/2$). The most general analysis of such a pump would require a cascade geometry in which figures 1(a) and 1(b) were *projections* of the geometry of a meridional surface (figure 2, section (Mbba)) onto a cylindrical surface and onto a plane perpendicular to the axis, respectively. (Note that the β_b marked in figure 1(b) is not appropriate when that diagram is used as a projection). We shall not attempt such generality here; rather, we observe that the meridional surface in many machines is close to conical. Denoting the inclination of the cone to the axis by ϑ, we can use equation (Mbbb4) to obtain an expression for the solidity of a simple cascade in this conical geometry,

$$s = Z_R \ell n \left(R_2/R_1\right) / 2\pi \sin \beta_b \sin \vartheta \tag{Mbbb7}$$

Clearly, this includes the expressions (Mbbb5) and (Mbbb6) as special cases.