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LINEARIZED DYNAMICS OF BUBBLY AND CAVITATING FLOWS IN CYLINDRICAL DUCTS

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ABSTRACT

The present work investigates the dynamics of the three-dimensional, unsteady flow of a bubbly mixture in a cylindrical duct subject to a periodic pressure excitation at one end. The results are then applied to the case of the idealized pressure excitation generated by the rotor stage of a turbomachine with the objective of understanding the dynamics of an inlet or discharge duct filled with bubbly liquid. The flow displays various regimes (subsonic, supersonic and super-resonant), with radically different propagation characteristics. Depending on the dispersion of the gaseous phase in the bubbly mixture and the angular speed of the turbomachine, the dynamic effects due to the bubble response can be significant, and the flow no longer behaves as a homogeneous barotropic fluid, as is commonly assumed. Examples are presented to illustrate the influence of various flow parameters.

1. INTRODUCTION

The dynamics of liquid/gaseous mixtures in ducts are relevant to a number of technological applications. For example, potentially dangerous instabilities can occur in pumping systems operating with bubbly or cavitating liquids. Typical in this respect, but in no way unique, are modern cryogenic liquid propellant rockets where, for obvious weight limitations, the propellant storage pressure is kept close to the saturation value. This inevitably increases the danger of vaporization, which routinely occurs as cavitation in the propellant feed turbopumps, but often extends also to the supply lines, with the development of a bubbly two-phase mixture. In this case, the highly increased compliance of the flow makes it quite susceptible to fluid dynamic oscillations and can be responsible for the onset of operational instabilities of the turbopump similar to rotating stall and surge phenomena commonly observed in compressors (Brennen, 1994). In turn, when coupled with turbomachine operation or thrust generation through flow asymmetries or propellant flux perturbations, these oscillations often develop into potentially catastrophic resonant phenomena, ranging from rotordynamic instabilities of the turbopumps to the onset of POGO instabilities of the entire propulsion system (Rubin 1966, Zielke 1969). The occurrence and, consequently, the prediction and control of these phenomena are crucially dependent on the unsteady dynamic response of the two-phase liquid/gaseous mixture in the flow lines. The reaction of a large number of bubbles, with non-negligible inertia, dissipation, and compliance, to the pressure in the mixture can lead to significant changes of the flow velocity, and therefore of the same pressure field that is ultimately responsible for driving the bubble response. Even at relatively low void fractions, this complex interaction mechanism drastically modifies the propagation of disturbances in the bubbly mixture and the spectrum of the internal oscillations of the flow.

This paper reports on an extension of previous research (d'Agostino and Brennen 1983, 1988, 1989, d'Agostino, Brennen and Acosta 1988) on the dynamics of bubbly and cavitating flows. In particular it makes reference to the paper by d'Agostino, Brennen and Acosta (1983) in which the expression for the natural frequency of a cloud of bubbles was first derived. The linear perturbation approach used in those previous analyses is applied to the more complex case of the three-dimensional unsteady flow of a bubbly mixture in a cylindrical duct subject to a periodic pressure excitation at one end. Despite the inherent limitations of the linear approximation, we are confident that this analysis illustrates some of the dynamic properties and fundamental phenomena of real bubbly liquids and contributes to the understanding of the flow instabilities occurring in several important engineering applications.

2. BASIC EQUATIONS AND LINEARIZATION

We address the problem of the simultaneous solution of the fluid dynamic equations for the two phases with the relevant interaction terms. For a detailed discussion of the governing equations the reader is referred to the paper by d'Agostino and Brennen (1988).

The continuity and momentum equations of the two phases are linearized to the first order in time-harmonic fluctuations (indicated by the tilde) of frequency $\omega \geq 0$ and complex representations:

$$u = \hat{u}e^{-i\omega t} \ , \ v = \hat{v}e^{-i\omega t} \ , \ p - p_o = \hat{p}e^{-i\omega t} \ ,$$

$$R - R_o = \hat{R}e^{-i\omega t} \text{ and } \beta - \beta_o = \hat{\beta}e^{-i\omega t}$$

from their unperturbed values (subscript o). In the above relations u is the fluid velocity, p is the pressure, R is the bubble radius and β is the bubble concentration per unit liquid volume. The same approach applied to the momentum and energy conservation equations for a bubble containing a perfect gas of uniform properties leads to modelling each individual bubble as an harmonic oscillator (d'Agostino and Brennen 1988, Prosperetti 1984). This procedure yields the following Helmholtz equation for $\hat{p}$:

$$\nabla^2 \hat{p} + k^2(\omega)\hat{p} = 0$$

with the free-space wave number ,K, determined by the dispersion relation:

$$\frac{1}{c_M^2(\omega)} = \frac{k^2(\omega)}{\omega^2} = \frac{\left[\frac{1}{c_{Mo}^2}\left(\frac{\omega_{Bo}^2(1+i\omega R_o/c)}{\omega_B^2 - \omega^2 - i\omega 2\lambda}\right) + \frac{(1-\alpha)^2}{c^2}\right]}{\left(1 - \frac{2\alpha(1-\alpha)}{1 - 9\mu/i\omega\rho R_o^2}\right)}$$

Here $c_M(\omega)$ is the complex and dispersive (frequency dependent) speed of propagation of an harmonic disturbance of angular frequency ω in the free bubbly mixture, $c = (dp/d\rho)^{1/2}$ is the speed of sound in the liquid, α is the void fraction, μ is the viscosity of the liquid and ρ its density; λ is the effective damping coefficient (Prosperetti, 1984) while

$$\omega_{Bo}^2 = \frac{3p_{Bo}}{\rho R_o^2} - \frac{2S}{\rho R_o^3} \quad \text{and} \quad c_{Mo}^2 = \frac{\omega_{Bo}^2 R_o^2}{3\alpha(1-\alpha)}$$

are, respectively, the natural frequency of oscillation of a single bubble at isothermal conditions in an unbounded liquid with surface tension S (Plesset and Prosperetti 1977, Knapp *et al.* 1970) and the low-frequency sound speed in a free bubbly flow with incompressible liquid ($\omega \to 0$ and $c \to \infty$).

3. DYNAMICS OF A BUBBLY FLOW IN A CYLINDRICAL DUCT

We address the problem of a three-dimensional unsteady flow in a cylindrical duct of length L and radius a, with rigid walls and arbitrary pressure excitation at the entrance, $x = 0$. We first examine the simpler case of a semi-infinite duct ($L \to +\infty$) whose relevant boundary conditions are $\hat{u}_r|_{r=a} \approx \partial\hat{p}/\partial r|_{r=a} = 0$, together with the radiation condition at $x = 0$, the regularity of the solution on the centerline, and its periodicity in the azimuthal direction. Then a duct of finite length is examined and the radiation condition is substituted by the appropriate boundary condition at $x = L$. As an example, we consider the case of a duct connected to a constant pressure reservoir, so that $\hat{p}|_{x=L} = 0$. By standard methods (Lebedev, 1965), the separable solutions (normalized, for convenience, at $x = 0$) of the above problems are respectively found to be:

$$\tilde{p}_{m,n}(\omega) \approx J_m\left(\frac{\alpha_{m,n} r}{a}\right)\exp\{i(\pm m\vartheta - \omega t + x k_x)\}$$

$$\tilde{p}_{m,n}(\omega) \approx J_m\left(\frac{\alpha_{m,n} r}{a}\right) \exp\{i(\pm m\vartheta - \omega t)\} \frac{\sin[(L-x)k_x]}{\sin[Lk_x]}$$

(or their complex conjugates), where J_m is the Bessel function of the first kind of integer order $m \geq 0$, $\alpha_{m,n}$ is the n-th non-negative root of $J_m'(z)=0$, the axial wave number is defined as $k_x = \sqrt{k^2(\omega) - \alpha_{m,n}^2/a^2}$ and the principal branch of the complex square root is chosen. We consider, in particular, the solution for the idealized excitation generated by a turbomachine with N blades (angular speed Ω) located at the duct entrance ($x=0$). Neglecting, for simplicity, the azimuthal component of the blade force, the pressure excitation is $2\pi/N$-periodic in the rotating angular coordinate $\vartheta' = \vartheta - \Omega t$ with assigned, Fourier decomposable distribution:

$$P(r,\vartheta') = \sum_{s=-\infty}^{\infty} c_s(r) e^{isN\vartheta'}$$

where:

$$c_s(r) = \int_{-\pi/N}^{\pi/N} P(r,\vartheta' t) e^{-isN\vartheta'} d\vartheta'$$

and, from the momentum balance of the duct fluid, $c_0 = 0$. Here s is the harmonic index of the Fourier decomposition, sN is the azimuthal mode number and $sN\Omega$ is the blade excitation frequency. Exploiting the linear nature of the problem, the corresponding solution is formally expressed, both for the finite and infinite length duct, by the Fourier-Bessel series:

$$\tilde{p} = \sum_{s=-\infty}^{\infty}\sum_{n=0}^{\infty} a_{sN,n} \frac{\tilde{p}_{|sN|,n}(|sN\Omega|)}{\bar{\tilde{p}}_{|sN|,n}(|sN\Omega|)}$$

with the upper (lower) conjugate solution valid for $s>0$ ($s<0$) and coefficients:

$$a_{0,0} = \frac{2}{a^2}\int_0^a r c_0(r)\,dr = 0$$

$$a_{sN,n} = \frac{2\int_0^a r c_s(r) J_{|sN|}\left(\alpha_{|sN|,n} r/a\right) dr}{a^2\left(1 - s^2N^2/\alpha_{|sN|,n}^2\right) J_{|sN|}^2\left(\alpha_{|sN|,n}\right)} \quad \text{for } s,n \neq 0,0$$

The remaining variables are readily obtained from the linearized governing equations. The entire flow has therefore been determined in terms of the material properties of the two phases, the geometry of the duct, the nature of the excitation, and the assigned quantities: R_o, p_o and α.

4. RESULTS AND DISCUSSION

To illustrate the phenomena manifest in this solution we choose a particular problem involving a duct of radius $a=0.15$m and length $L=1$ m, containing air bubbles ($R_o = 0.001$ m, $\gamma = 1.4$, $\chi_G = 0.0002$ m^2/s) in water ($\rho = 1000$ kg/m^3, $\mu = 0.001$ Ns/m^2, $S = 0.0728$ N/m, $c = 1485$ m/s) at atmospheric pressure $p_o = 10^5$ Pa. In addition, in most cases the parameter $3\alpha(1-\alpha)a^2/R_o^2$ is assigned and the void fraction α is determined accordingly.

We now consider in some detail the general features of the propagation of disturbances through the duct. Note, first of all, that the solution for a semi-infinite duct is simply harmonic in ϑ, t and, in complex sense, also in x, while its behavior in the radial direction is expressed by Bessel functions of integer order m, scaled through the factors $\alpha_{m,n}/a$ in order to satisfy the kinematic boundary condition at the duct radius. The first few radial mode shapes are illustrated in Figures 1 and 2 for $m=0,1$. In particular, the solution for $m=n=0$ corresponds to plane axial waves. The undamped axial mode shapes are purely sinusoidal. They are modified, in a complex way, by the inclusion of damping through the axial wave number, whose real and imaginary parts respectively determine the axial wave length and attenuation rate. The axial modes shape will then be significantly dependent on the flow regime.

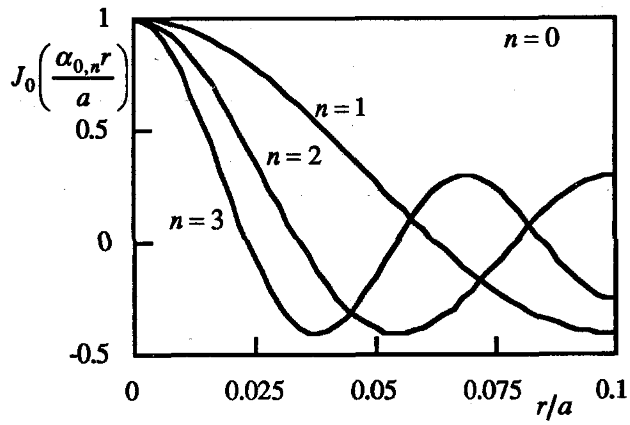

Figure 1. Radial mode shapes $J_0(\alpha_{0,n} r/a)$ as a function of r/a for the fundamental azimuthal harmonic ($m=0$) and several radial mode numbers $n=0,1,2,3$.

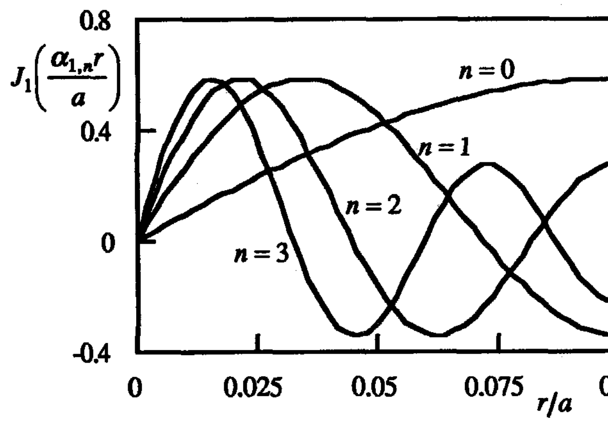

Figure 2. Radial mode shapes $J_1(\alpha_{1,n}r/a)$ as a function of r/a for the first azimuthal harmonic ($m=1$) and several radial mode numbers $n=0,1,2,3$.

The real and imaginary parts of the axial wave number are generally different from the free-space values $\mathrm{Re}(k)$ and $\mathrm{Im}(k)$ because of the presence of the duct boundaries, except for the simple case of plane axial waves where $\alpha_{m,n}=\alpha_{0,0}$ vanishes. The behavior of k_x is more readily illustrated in the absence of damping (when the argument of the square root is real) as shown in Figures 3 and 4 for the sample case of $m=1$ and $n=0$ ($\alpha_{1,0}=1.8412$). Then the axial wave number is either real or purely imaginary depending on the sign of $k^2(\omega)-\alpha_{m,n}^2/a^2$, with a first regular transition at the cut-off frequencies:

$$\omega_{m,n}^2=\omega_{Bo}^2\bigg/\left(1+\frac{3\alpha(1-\alpha)a^2/R_o^2}{\alpha_{m,n}^2}\right)$$

and a second, singular transition at the natural frequency ω_{Bo} of an individual bubble in an infinite liquid (bubble resonance condition), where $k^2(\omega)$ has a simple pole. Notice that the cut-off frequencies are never greater than ω_{Bo} and, for any given azimuthal harmonic m, increase with the radial mode number n. In addition, the cut-off frequencies depend on the phase dispersion parameter $3\alpha(1-\alpha)a^2/R_o^2$ as shown in Figure 5 for several values of m in the simple case of $n=0$. Note that a similar dispersion parameter occurs in all bubble cloud analyses (d'Agostino and Brennen, 1983, 1988,1989; Kumar and Brennen 1990). When this parameter is of order unity or larger, the lower cut-off frequencies are significantly smaller than the bubble resonance frequency; when it is less than unity the cut-off frequencies are contained in a narrow range only slighlty below bubble resonance frequency.

As a result of the higher cut-off frequencies for the higher radial modes, appreciable wave-like propagation of m-lobed azimuthal excitation sources takes place when the frequency, ω, falls between the cut-off frequency $\omega_{m,0}$ of the fundamental radial mode ($n=0$) and the bubble resonance frequency $\omega=\omega_{Bo}$. When applied to the pressure field generated by a turbomachine with N blades and angular speed Ω, this condition implies:

$$\alpha_{sN,0}^2\le k^2(sN\Omega)a^2=\frac{s^2N^2\Omega^2}{c_M^2(sN\Omega)}a^2=s^2N^2M_{sN}^2$$

or

$$M_{sN}\ge\frac{\alpha_{sN,0}}{sN}$$

where $\alpha_{sN,0}/sN$ is the cut-off value of the blade tip Mach number $M_{sN}=\Omega a/c_M(sN\Omega)$ relative to the propagation speed of the harmonic disturbances with angular frequency $\omega=sN\Omega$ actually induced by the rotation of the turbomachine.

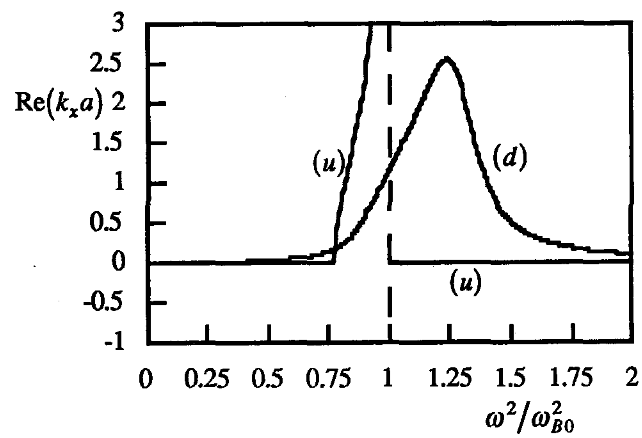

Figure 3. Real part $\mathrm{Re}(k_x a)$ of the normalized axial wave number as a function of the square of the reduced frequency, ω/ω_{Bo}, with (d) and without (u) damping for the fundamental radial mode of the first azimuthal harmonic ($m=1$, $n=0$, $\alpha_{m,n}=\alpha_{1,0}=1.8412$) and $3\alpha(1-\alpha)a^2/R_o^2=1$.

The values of $\alpha_{sN,0}/sN$ are always slightly supersonic and approach unity as the azimuthal mode number sN tends to infinity. Therefore, in more familiar terms, effective propagation of the disturbances generated by a turbomachine operating with bubbly flow in a cylindrical duct is limited to the excitation from supersonic rotors not exceeding the bubble resonance condition ($sN\Omega\le\omega_{Bo}$). This phenomenon is in line with well-established results for compressible non-dispersive barotropic fluids (Tyler & Sofrin 1962, Benzakein 1972), and may have important implications with reference to the onset and stability of rotating cavitation in the suction lines of pumping systems operating with bubbly or cavitating flows.

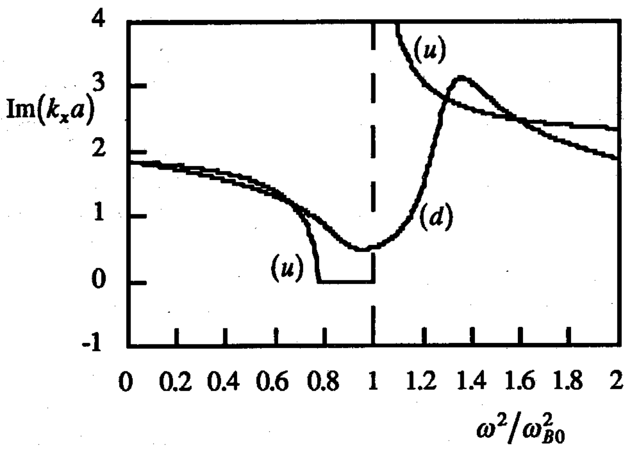

Figure 4. Imaginary part $\mathrm{Im}(k_x a)$ of the normalized axial wave number as a function of the square of the reduced frequency, ω/ω_{Bo}, with (d) and without (u) damping for the fundamental radial mode of the first azimuthal harmonic ($m=1$, $n=0$, $\alpha_{m,n}=\alpha_{1,0}=1.8412$) and $3\alpha(1-\alpha)a^2/R_o^2=1$.

The inclusion of damping (see Figures 3 and 4) makes the axial wave number k_x (and therefore also the cut-off frequency $\omega_{m,n}$) complex and eliminates the singularity at $\omega=\omega_{Bo}$, thereby blurring to some extent the transitions between the three propagation regimes. Energy dissipation also increases the typical frequencies of the flow, effectively damping the higher resonant modes.

From the relevant expression of $\tilde{p}$ note that free oscillations ($a_{m,n}=0$) of bubbly flows in finite-length ducts can only occur when $\sin[k_x(\omega)L]=0$ (where l is a positive integer), a condition that, together with the dispersion relation, determines the natural frequencies $\omega_{m,n,l}$ and mode shapes $\tilde{p}_{m,n,l}$. The natural frequencies $\omega_{m,n,l}$ always lie between the corresponding cut-off frequencies $\omega_{m,n}$ and ω_{Bo}, increase with the axial mode number l, and converge to ω_{Bo} as $L\to+\infty$. Just like the cut-off frequencies, the natural frequencies decrease with the parameter $3\alpha(1-\alpha)a^2/R_o^2$, as illustrated in Figure 6 for the lower values of l in the simple case of $n=0$ (fundamental radial mode). They can be appreciably smaller than ω_{Bo} when this parameter is of order unity or larger.

The relative amplitudes of the pressure and bubble radius oscillations in a semi-infinite duct are shown in Figures 7 and 8 as a function of the normalized frequency for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=0,1,2,3$) with $3\alpha(1-\alpha)a^2/R_o^2=1$, corresponding to $a=0.15$m and $R_o=0.001$ m.

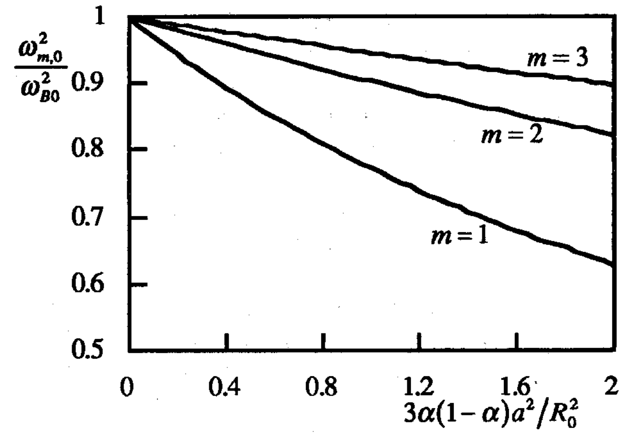

Figure 5. Normalized cut-off frequency $\omega_{m,0}^2/\omega_{Bo}^2$ as a function of the phase dispersion parameter $3\alpha(1-\alpha)a^2/R_o^2$ for the fundamental radial mode ($n=0$) of the lower azimuthal harmonics ($m=1,2,3$).

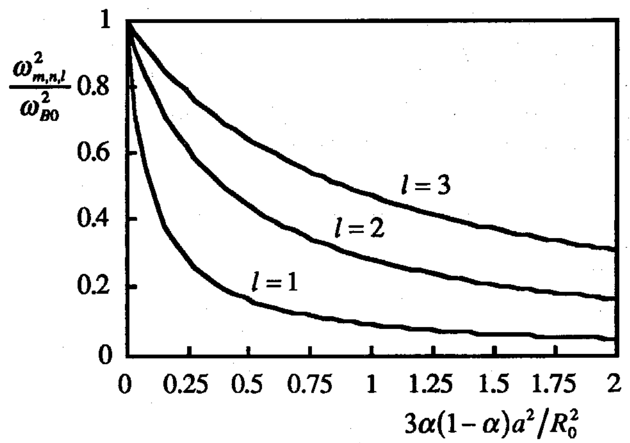

Figure 6. Normalized natural frequency $\omega_{m,n,l}^2/\omega_{Bo}^2$ as a function of the phase dispersion parameter $3\alpha(1-\alpha)a^2/R_o^2$ for the fundamental radial mode ($n=0$) of the first azimuthal harmonic ($m=1$) and axial mode numbers ($l=1,2,3$).

The corresponding results for a finite-length duct are also shown in Figures 9 and 10. Appreciable oscillations are only observed for the fundamental and first azimuthal harmonics. Oscillations corresponding to the duct natural frequencies , as expected, are roughly concentrated in the frequency range from the lower cut-off frequency $\omega_{m,0}$ (the cut-off frequency for plane axial wave is zero) to the bubble resonance frequency ω_{Bo}, as a consequence of the rather large value of the phase dispersion parameter $3\alpha(1-\alpha)a^2/R_o^2$. Notice the amplitude of the flow oscillations in the finite-length duct due to the resonance peaks, which are clearly absent in the case of the semi-infinite duct. Further computation has shown that higher azimuthal harmonics start showing appreciable oscillations when the length of the duct is decreased. The same trend has been observed when increasing the radius of the chamber for a given duct length. The observed behaviour shows the key role played by the non-dimensional parameter a/L in the dynamics of a bubbly flow in a cylindrical duct.

Finally it is worth mentioning that, because of the functional dependence of the natural frequencies, the peaks corresponding to the same natural modes of oscillation have been observed to occur at different frequencies moving towards the origin at higher values of $3\alpha(1-\alpha)a^2/R_o^2$. Furthermore the maximum amplitudes of the bubble radius decreases with the phase dispersion parameter owing to the greater compliance of the flow. This phenomenon is consistent with experimental observations in travelling bubble cavitating flows (Arakeri & Shanmuganathan 1985; Marboe, Billet & Thomson 1986; Arakeri & Shanmuganathan 1985; Ceccio & Brennen 1990) and, for the particular geometry under consideration, may have significant implications on noise generation, cavitation damage in pumping systems operating with bubbly and cavitating flows

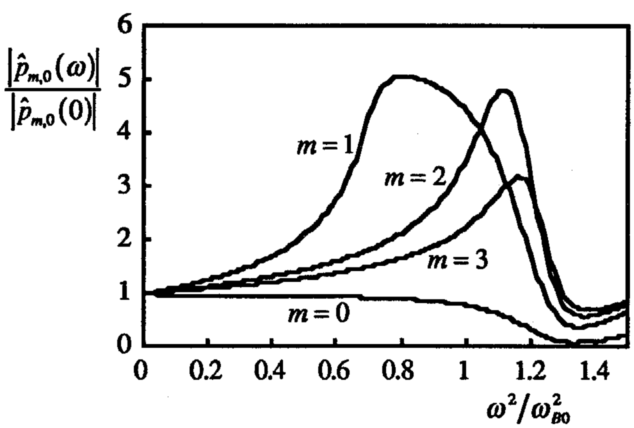

Figure 7. Normalized amplitude of the pressure oscillations $|\hat{p}_{m,0}(\omega)|/|\hat{p}_{m,0}(0)|$ as a function of the reduced frequency, ω/ω_{Bo}, for the fundamental radial mode ($n=0$) of the lowest azimuthal harmonics ($m=0,1,2,3$) in a semi-infinite duct with $3\alpha(1-\alpha)a^2/R_o^2=1$.

5. CONCLUSIONS

The results of this study reveal a number of important effects occurring in bubbly and cavitating flows in cylindrical ducts as a consequence of the strong coupling between the local dynamics of the bubbles and the global behavior of the flow. The propagation of disturbances along the duct is significantly modified by the large reduction of the sonic speed, which becomes both complex (dissipative) and dispersive (frequency dependent). Additional modifications are introduced by the the boundaries, which determine the excitation modes and their cut-off frequencies, and, in finite length ducts, the natural frequencies and shapes of free motions. Appreciable wave-like propagation of each excitation mode along the duct is limited to the frequency range between cut-off and bubble resonance conditions, and, except for plane waves, is characteristic of supersonic (but sub-resonant) flows, as defined by d'Agostino, Brennen and Acosta (1988). In finite-length ducts, the same frequency range also contains the infinite set of natural frequencies of the resonant modes. The different propagation properties of subsonic, supersonic and super-resonant flows are due to the relative importance of pressure and inertial forces in the bubble dynamics at different excitation frequencies, as already outlined in previous papers (d'Agostino and Brennen, 1988).

In duct flows excited by a turbomachine, only the perturbations from supersonic rotors propagate effectively and are potentially capable of becoming self-sustained when effectively reflected by the downstream boundary condition. Given the low sonic speed of bubbly mixtures, the cut-off conditions can readily be exceeded in fast, high-performance turbopumps when the phase dispersion parameter is sufficiently high. This phenomenon is therefore potentially relevant to surge-like auto-oscillations and rotating cavitation instabilities in pumping systems operating with bubbly flows (Brennen, 1994).

Viscous and thermal dissipation in the dynamics of the bubbles usually dominate the other contributions to the damping. In the present context, energy dissipation eliminates the resonance singularities, effectively damping the highest natural modes. The spectral response of the flow is therefore dominated by the lowest resonant modes, whose amplitudes and distribution crucially depend on the phase dispersion parameter $3\alpha(1-\alpha)a^2/R_o^2$. The increase of this parameter causes a substantial reduction of the bubble response peaks owing to the greater compliance of the flow, and a parallel reduction in the corresponding frequencies.

The length to radius ratio of the duct also plays an important role in the dynamics of this kind of flow.

Higher modes display stronger oscillations as the aspect ratio L/a of the duct increases and the peak frequencies are strongly shifted towards lower values of ω/ω_B.

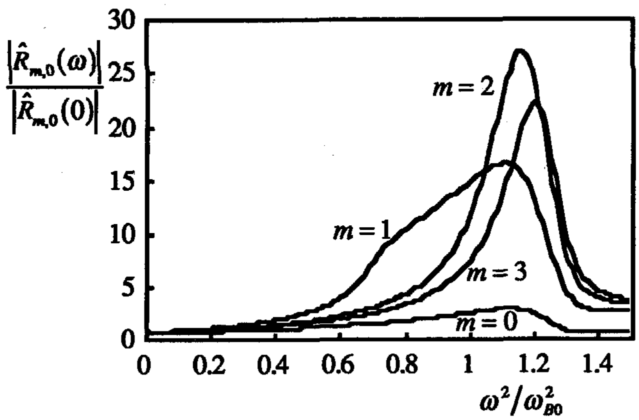

Figure 8. Normalized amplitude of the bubble radius oscillations $|\hat{R}_{m,0}(\omega)|/|\hat{R}_{m,0}(0)|$ as a function of the square of the reduced frequency, ω/ω_{B0}, for the fundamental radial mode $(n=0)$ of the lowest azimuthal harmonics $(m=0,1,2,3)$ in a semi-infinite duct with $3\alpha(1-\alpha)a^2/R_o^2=1$ and $3\alpha(1-\alpha)L^2/R_0^2=44.4$.

The present theory has been derived under fairly restrictive linearization assumptions and therefore is not expected to provide a quantitative description of unsteady bubbly flows in cylindrical ducts, except in the acoustical limit. Bubble radius perturbations are often large in technical applications, and the flow velocity can be comparable to the sound speed in the bubbly mixture. Therefore the most crucial limitations of the present theory are the linearization of the bubble dynamics and the neglect of the mean flow velocity, while the assumption of small velocity perturbations is likely to be more widely justified.

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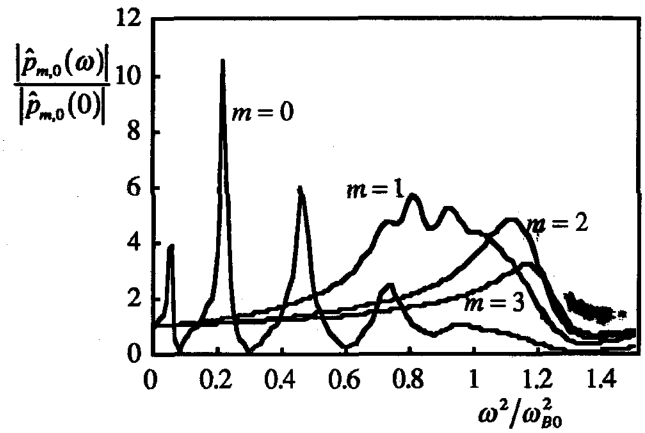

Figure 9. Normalized amplitude of the pressure oscillations $|\hat{p}_{m,0}(\omega)|/|\hat{p}_{m,0}(0)|$ as a function of the square of the reduced frequency, ω/ω_{B0}, for the fundamental radial mode $(n=0)$ of the lowest azimuthal harmonics $(m=0,1,2,3)$ in a finite-length duct with $3\alpha(1-\alpha)a^2/R_o^2=1$ and $3\alpha(1-\alpha)L^2/R_0^2=44.4$.

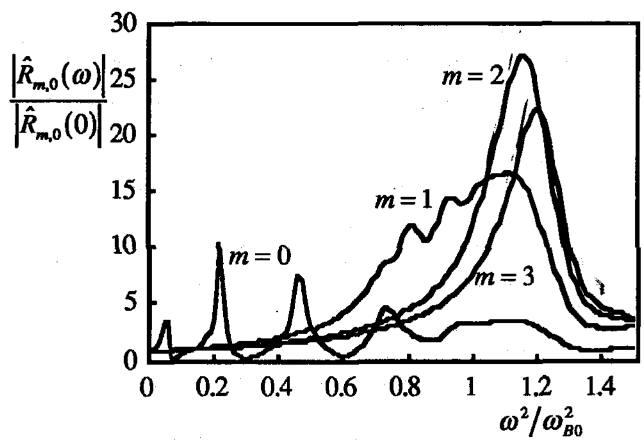

Figure 10. Normalized amplitude of the bubble radius damped oscillations $|\hat{R}_{m,0}(\omega)|/|\hat{R}_{m,0}(0)|$ as a function of the square of the reduced frequency, ω/ω_{B0}, for the fundamental radial mode $(n=0)$ of the lowest azimuthal harmonics $(m=0,1,2,3)$ in a finite-length duct with $3\alpha(1-\alpha)a^2/R_o^2=1$ and $3\alpha(1-\alpha)L^2/R_0^2=44.4$.

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