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Chute Flows of Granular Material: Some Computer Simulations

A computer simulation has been developed to describe unidirectional flows of granular materials. Results are presented for a simulation of the two-dimensional flow of disks or cylinders down an inclined plane or chute. Velocity and solid fraction profiles were measured from the simulated systems and compared with theoretical analyses and are compared with the limited experimental results now available. The behavior is shown to be critically dependent on a third field quantity—the "granular temperature"—a measure of the kinetic energy contained in the random motions of the particles.

1 Introduction

In recent years there has been a significant increase of interest in the detailed mechanics of granular material and considerable progress has been made on understanding their behavior within the limiting flow regimes. The state of knowledge has been recently reviewed by Spencer [20], Mroz [13], and Savage [17]. Spencer and Mroz concentrated their review on flows with extremely low strain rates: the so-called "quasistatic" regime. Of greater relevance to the design of industrial chutes and hoppers are the high strain-rate flows in the "rapid flow" or "grain inertia" regime reviewed by Savage [17]. This latter regime encompasses any flow in which the particle interactions can be modeled as instantaneous collisions. All of these regimes are defined for flows in which the interstitial fluid plays a negligible role in determining the rheology.

Research on granular material flows has been severely hampered by the lack of reliable, nonintrusive instrumentation to measure precisely the velocity, solid fraction, and random particle motion within such flows. An alternative approach is to utilize the kind of computer simulations that have been quite successful in the analogous problems in molecular gas dynamics (e.g., [3, 23]).

The purpose of the present paper is to present results from computer simulations of the flow of dry granular material down inclined chutes or channels and to compare the results of these calculations with existing experimental measurements and theoretical models. The utility of such computer simulations is twofold. First, they provide insights and detailed results that can be used to suggest appropriate assumptions for the theoretical models and to validate existing models. At the present time this is best accomplished using simple shear, or Couette flows. Simulations of such flows by the present methods are reported in other papers (Campbell [4], Campbell and Brennen [5]) where the variation of velocity distributions (deviations from Maxwellian distributions at higher solid fractions), collision angle distributions, and other statistical information is provided.

A second objective of the computer simulations is to compare the results with experimental measurements to establish appropriate models of the mechanics of particle/particle and particle/wall interactions. Much of the existing measurements of the kind on shear flows come from experiments on granular material flow down inclined (or vertical) channels or "chutes" (see references of Section 5). Thus the objective of this paper is to briefly report on some comparisons between the results of computer simulation and the existing data for flows down inclined chutes. Further details can be found in Campbell [4].

Parenthetically one might add that a third use of computer simulations might be to explore the effects of additional forces such as those due to the interstitial fluid and electrostatic effects. For simplicity, such effects are not included at the present state of the development of the simulation. Discussion of interstitial fluid effects can be found in other recent works (Savage [17]).

Other computer models of granular material flows include the work Cundall [6], Davis and Deresiewicz [8], Cundall and Strack [7], Trollope and Berman [21], and Walton [22]. Most of these are directed toward the simulation of slower flows and smaller deformations. However, in his original work Cundall [6] did extend his methods to some higher speed but transient flows such as rockfalls and the emptying of a hopper. Also the versatility of Walton's [22] computer program is readily apparent from the excellent movies that the Lawrence Livermore Laboratory has produced. However, to our knowledge, none of the existing models have been used to produce continuous "steady" flows which could be used for basic rheological studies. The intent of the present model is to

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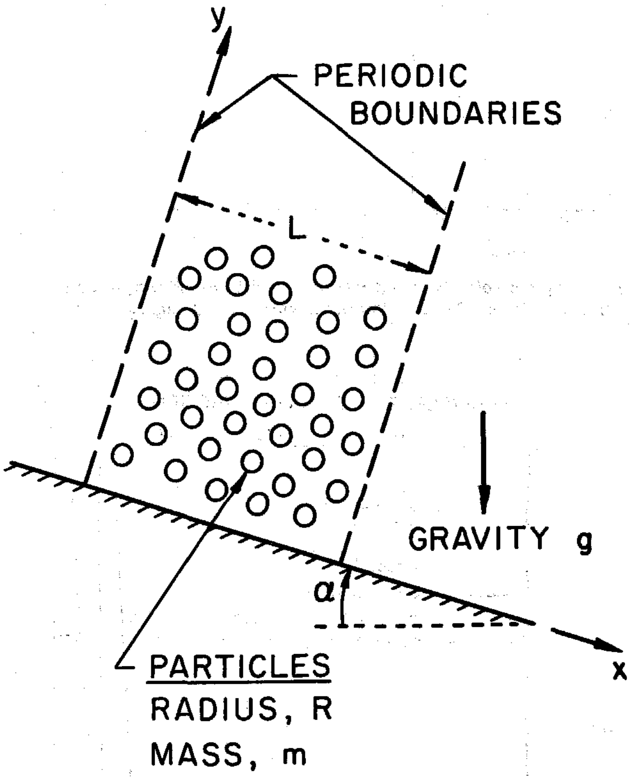

Fig. 1 Schematic of chute flow simulation

minimize the complexity of the geometry and the interactions so that steady flows of sufficiently long duration for rheological analysis would be produced. Thus the simulations are just two-dimensional and the particles are circular cylinders. Though extension of the simulation to the third dimension is possible, the necessary computer costs would be considerably greater.

2 Computer Simulation

The present method for the computer simulation of granular material flows has been documented elsewhere (Campbell [4], Campbell and Brennen [5]) and will only briefly be described in the present paper. Two-dimensional *unidirectional flows of inelastic circular cylinders* are followed mechanistically. Though both Couette flows and chute flows have been examined, the present paper is confined to discussion of the simulations of chute flow.

The simulation is initiated by placing a number of cylindrical particles in a control volume bounded by the chute bottom and two perpendicular "periodic" boundaries (see Fig. 1). A particle passing out of the control volume through one of the periodic boundaries immediately reenters the control volume through the other periodic boundary at the same relative height above the chute bottom and with the same instantaneous velocity. This greatly increases the computational efficiency of the simulation by restricting the number of particles to that originally placed in the control volume, but has the side effect of eliminating all gradients in the flow direction (x direction, Fig. 1). The number of particles originally placed in the control volume will also determine the eventual depth of the flow. The separation, L, between the periodic boundaries was adjusted until further doubling of this length (as well as the number of particles) had little effect on the results. For the flows described here values of L/R (R is the particle radius) of about 10 were found to be sufficient for this purpose (see Fig. 2).

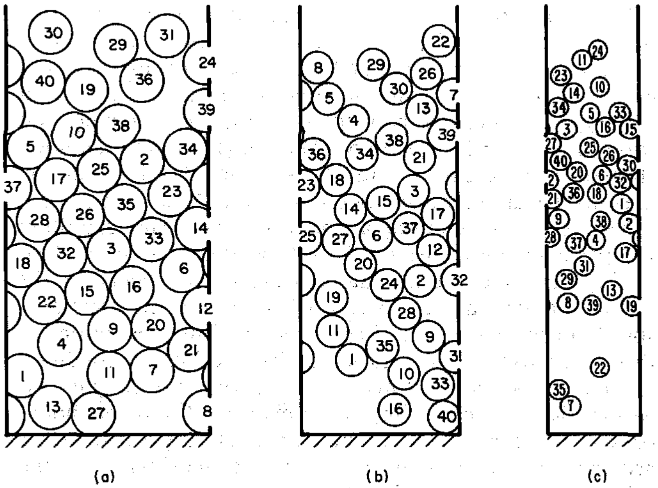

Fig. 2 Typical snapshots from the chute flow simulations for inclinations of (a) $\alpha = 20$ deg, (b) $\alpha = 30$ deg, and (c) $\alpha = 40$ deg

All simulations were begun with a randomly perturbed square lattice array of particles in the control volume and randomly chosen instantaneous translation velocities (u, v, in the x, y directions of Fig. 1) and rotational velocity (ω). Consequently, it is convenient to refer to simulations by the dimensions of this initial array; for example, 10×4 is a simulation started with 10 rows of 4 particles.

The intent is to mechanistically follow the entire assemblage of particles, under the action of gravity (g), as it proceeds from collision to collision, until a final asymptotic state is reached in which the flow is steady in the sense of being invariant over long time scales. Details of the convergence toward such a state are described in Campbell [4] and will not be repeated here. The results presented here were run for as long as 200,000 collisions. Though the Couette flows described elsewhere required as little as 20,000 collisions for convergence of a typical 10×4 simulation, the present chute flows required considerably longer. Furthermore determination of convergence was clouded by the fact that most of the simulations exhibit the continuous fluctuations typical of small thermodynamic systems (Landau and Lifshitz [12]); this considerably increased the duration of "steady flow" necessary to perform meaningful evaluation of the flow properties.

In summary, it is believed that the simulations with chute *inclinations of 30 deg are well converged. However, those at* $\alpha = 20$ deg and $\alpha = 40$ deg may still have contained some small deceleration and acceleration, respectively, at the conclusion of the simulations.

Each individual particle/particle or particle wall collision is assumed to be instantaneous and the resulting departure velocities calculated by conventional means. The two closure conditions used were as follows. First the relative approach and departure velocity components parallel to the line of centers are related by a coefficient of restitution to represent the inelasticity of the collisions. Different coefficients (denoted respectively, by ϵ_p and ϵ_w) can be used for particle/particle and particle/wall collisions. Values of 0.6 and 0.8 were deemed appropriate as a result of experiments with typical polystyrene beeds and aluminum walls and most of the simulations used these values. Secondly, it is necessary to establish a frictional closure condition pertaining to the relative tangential velocities upon departure. Most of the simulations described in this paper employ a condition requiring zero relative tangential velocity of the contact points upon depature from a collision (Type A condition). In a real collision of finite duration, such closure condition would pertain provided the friction between the contact points was sufficient for relative tangential motion to cease at some time prior to departure. This is a reasonable approximation for particle/particle collisions as the relative tangential velocities at impact are generally small. However, for particle/wall collisions, the relative tangential velocities may be large and the Type *A* collisions may result in too much momentum transfer into particle rotation. (This rough particle condition should not be confused with the artificial closure condition used for rough molecule models of gases). A few simulations were also performed using a more artificial closure condition for particle/wall collisions. This Type *B* condition involved setting the relative tangential velocity of the *center* of the particle to zero upon depature while leaving the rotational speed unchanged from its incident value. The motivation for this alternative is found in the results; the Type *A* condition yielded slip at the wall and an alternative was sought that would yield no-slip. Many other choices of wall boundary condition are possible. At the very least, these two rather arbitrary alternatives demonstrate the sensitivity of the solution to the choice of particle/wall closure condition (see Section 6).

3 Presentation of Results

All flow properties were obtained as functions of distance, y, from the chute bottom by dividing the control volume into strips and computing the particle-weighted mean of the property in each strip (Campbell [4]) over a substantial length of time (up to 20,000 collisions). The periodic boundary conditions assure that there will be no change of mean properties in the x direction. The mean velocity, $\langle u \rangle$, and mean solid fraction, ν, were obtained in this way. Data is also presented for the "translational temperature" defined as the kinetic energy (divided by $\nu\rho_p/2$) associated with the random motions of the particles. The latter is obtained by subtracting the kinetic energy of the mean shear flow from the total instantaneous kinetic energy. Specifically the nondimensional translational temperature is defined as $(\langle u^2 \rangle + \langle v^2 \rangle - \langle u \rangle^2 - \langle v \rangle^2)/gR$, and is denoted in the figures as $(\langle u'^2 \rangle + \langle v'^2 \rangle)/gR$. This quantity is of particular interest from a rheological point of view (Savage [17]) because it is presently believed that the ratio of the typical velocity difference due to shear, $2Rd\langle u \rangle/dy$, to the typical random translation velocity, $(\langle u'^2 \rangle + \langle v'^2 \rangle)^{1/2}$ plays a central role in determining the constitutive behavior of the flowing granular media (Savage [19], Jenkins and Savage [10], Savage and Jeffrey [18]). The results presented here are all nondimensionalized using the intrinsic scales for length, R, for velocity, $(gR)^{1/2}$, and for mass, m (mass of a particle). It should also be noted that the only nongeometric parameters inputed to the simulation are the coefficients of restitution, ϵ_p and ϵ_w, and the ratio, β, of the square of the radius of gyration of the particles to the square of their radius. All the present simulations were run with $\beta = 1/2$ the value appropriate to cylinders.

4 Results of the Chute Flow Simulations

Three typical instantaneous snapshots of the arrangement of particles in the control volume are presented in Fig. 2 for flows at $\alpha = 20$, 30, and 40 deg, respectively. Note in particular that as the angle and therefore the velocity and shear rate of the flow is increased, the bulk of the flow appears to ride on a lower solid fraction layer of highly agitated particles.

Typical velocity, solid fraction, and temperature profiles for simulations with the Type *A* wall condition are presented in Fig. 3. The velocity profile has the same shape one would expect with a liquid except for a slip velocity at the wall amounting to about 50 percent of the maximum velocity. The solid fraction profile attains its maximum value in the center of the flow and exhibits a region of reduced density next to the solid bottom and a "saltated" region of lower density on the upper surface.

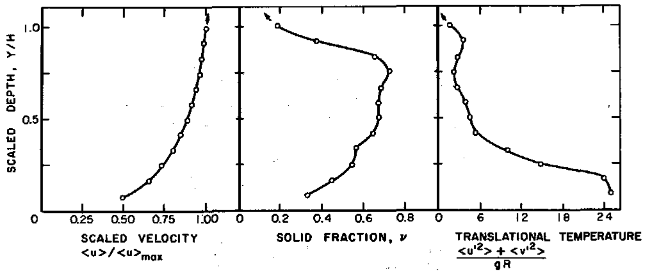

Fig. 3 Typical velocity, solid fraction, and translational temperature profiles for a 10 × 4 Type *A* simulation with α = 30 deg, ϵ_p = 0.6, ϵ_w = 0.8

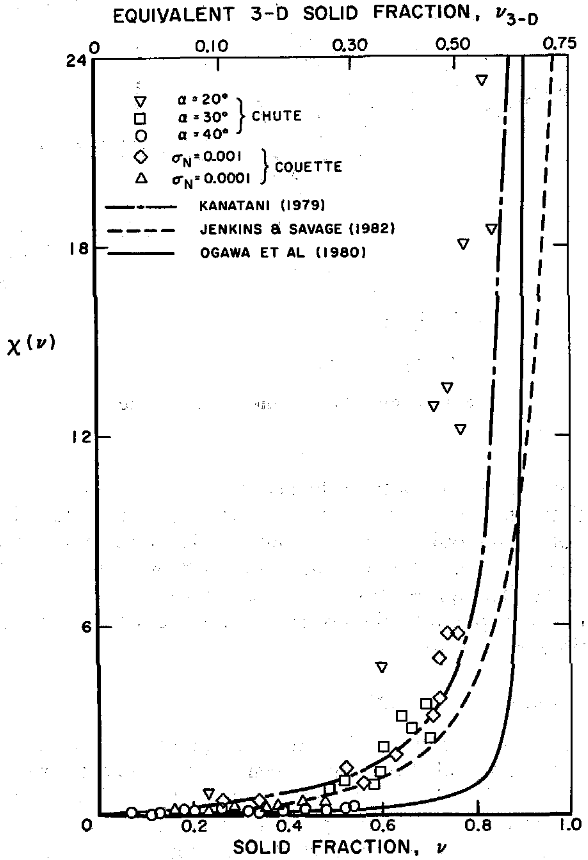

Fig. 4 Solid fraction dependence of the equation of state

The random kinetic energy of the particles, or temperature, is a by-product of the collisions between particles. Its magnitude depends on the magnitude of the relative velocity of the particles at collision, which will in turn be related to the local shear rate. (See Campbell [4], Campbell and Brennen [5].) Thus the largest temperature is found in the high shear zone next to the wall.

One might speculate that the low density region near the wall is a result of the high temperature in that region despite the overburden of the material. This suggests the existence of an "equation of state." Equations of state have been proposed by Jenkins and Savage [10], Ogawa et al. [15], and Kanatani [11]. All these have general form:

$$P = \chi(\nu)(\langle u'^2 \rangle + \langle v'^2 \rangle) \tag{1}$$

where $\chi(n)$ is a nonlinear function crudely similar in motivation to the high-density correction factor of the Van-

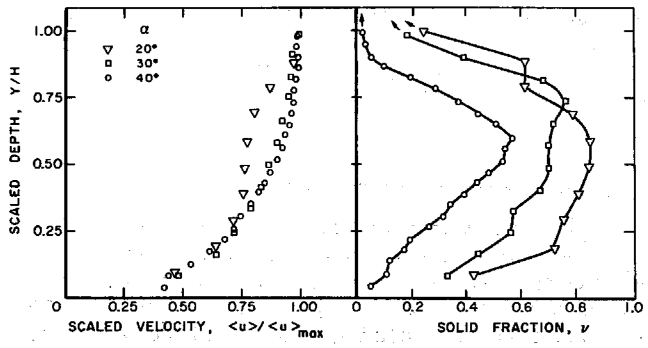

Fig. 5 Velocity and solid fraction profiles for different chute inclinations (α = 20, 30, and 40 deg) for Type *A* simulations (10 × 4) with ϵ_p = 0.6, and ϵ_w = 0.8

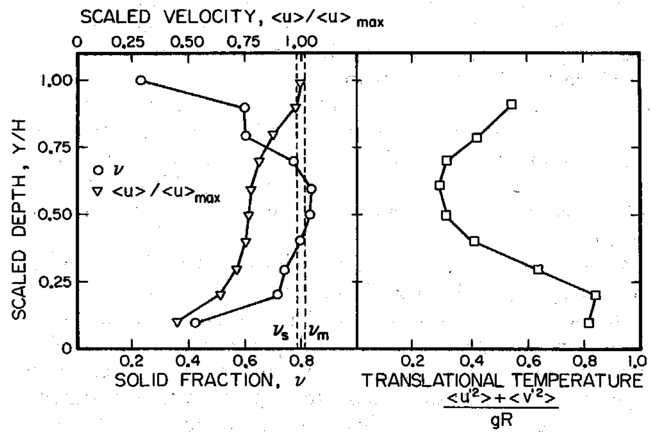

Fig. 6 Velocity, solid fraction, and translational temperature profiles for the α = 20 deg Type *A* simulation (10 × 4, ϵ_p = 0.6, ϵ_w = 0.8) showing plug flow region

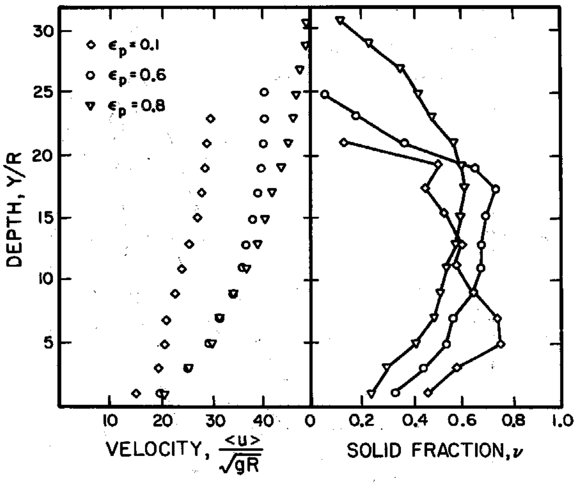

Fig. 7 Effect of particle/particle coefficient of restitution, ϵ_p on velocity and solid fraction profiles from Type *A* simulation (α = 30 deg, 10 × 4 simulation, ϵ_w = 0.8)

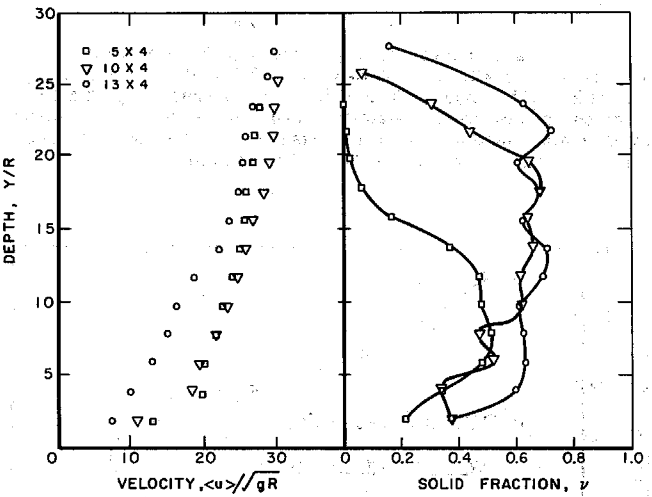

Fig. 8 Effect of flow depth on the velocity and solid fraction profiles of the α = 30 deg Type *A* simulation

der-Waal's equation of state. Values of $\chi(\nu)$, taken from the simulation by assuming p to the local static pressure due to the overburden are plotted in Fig. 4, and compared with the predictions of [10, 11, 14]. Some data from the simulations of Couette flows [4, 5] is also included in the figure. Note that the simulation yields values of χ that appear to asymptote to infinity near ν = 0.91, a value that corresponds to the maximum possible packing of disks. It also agrees qualitatively with the theories of Jenkins and Savage [10] and Kanatani [11] though the comparison is clouded by the fact that the asymptote in [10] occurs at ν = 1 whereas both Kanatani [11] and Ogawa [14] leave this asymptote unspecified; ν = 0.91 is used in plotting data from both these theories.

Velocity and solid fraction profiles for Type *A* simulations at chute inclination angles of 20, 30, and 40 deg are shown in Fig. 5. Note that the maximum solid fraction decreases as a function of inclination angle due to the generally higher temperatures generated by larger velocities.

Note also that the velocity profile for α = 20 deg deviates from that for α = 30 or 40 deg. This is due to the existence of a nonshearing plug in the center of the dense, slow moving flow, a phenomenon that did not occur at higher inclination angles. The data for α = 20 deg is repeated in Fig. 6 to further document this interesting example of the coexistence of a high-density unshearing plug and a lower-density shearing flow. Also indicated in Fig. 6 is the square packing solid fraction, ν_s, and the maximum solid fraction that permits the shearing of cylinders, ν_m (Campbell [4]). The solid fraction exceeds both these values in a central part of the flow and therefore the shear rate, $d\langle u\rangle/dy$ must be zero at least when $\nu > \nu_m$. However, the granular temperature of this plug is not zero. But in the absence of shear this temperature cannot be internally generated. Hence the temperature profile of Fig. 6 implies the conduction of fluctuating kinetic energy into the plug from the sheared regions both on top of and below the plug and the subsequent dissipation of this fluctuating kinetic energy by collisions within the matrix of the plug.

The typical data for different particle/particle coefficients of restitution, ϵ_p, is shown in Fig. 7. Lower values of ϵ_p tend to cause blunter, more plug-like velocity profiles with higher values of the solid fraction. The three simulations in Fig. 7 all have the same number of particles in identical control volumes (10 × 4 simulations). Consequently, the higher the value of ϵ_p, the more dilated and deeper the flow. Other simulations with different values of the particle/wall coefficient of restitution (ϵ_w) showed that this parameter had only a minor effect on the flow (Campbell [4]).

The effect of the flow depth is demonstrated in Fig. 8 using simulations with initial square lattice arrangements of 5 × 4, 10 × 4, and 13 × 4 particles and control volumes of the same width. The velocity variations across the depth tend to increase with increasing depth. Also the maximum solid fraction seems to approach a limiting value, which is first attained here in the 10 × 4 simulation; further increase in depth leads to an increase in the extent of this central region

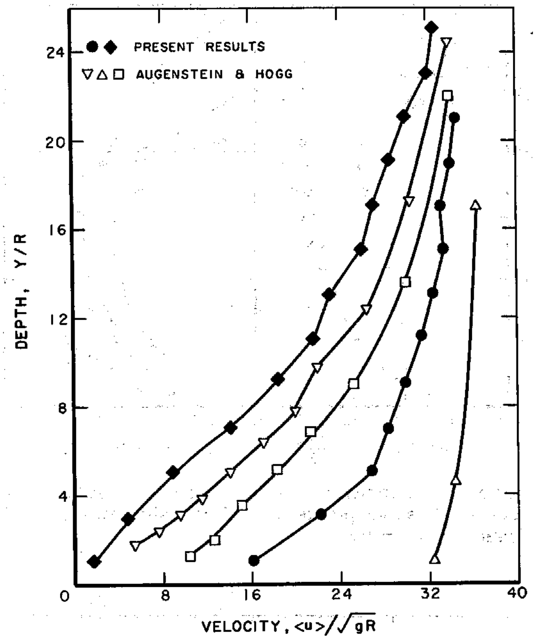

Fig. 9 Comparison of the velocity profiles from Type *A* (•) and Type *B* simulations (◆) with α = 40 deg, ϵ_p = 0.6, and ϵ_w = 0.8 (5 × 4 simulation) with profiles measured by Augenstein and Hogg [1]: △, 35 × 48 mesh sand on smooth stainless steel surface; □, 28 × 35 mesh sand on roughened with 65 × 100 mesh sand; and ▽, 35 × 48 mesh sand on surface roughened with 35 × 48 mesh sand

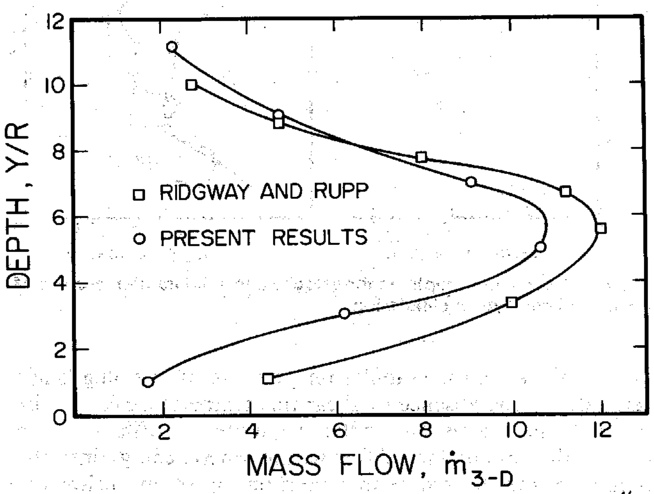

Fig. 10 Comparison of one of the mass flow rate ($\nu_{3D}\,\langle u\rangle/(gR)^{1/2}$) profiles of Ridgway and Rupp [15] with Type *A* simulation of the same flow (α = 40 deg, 4 × 4 simulation)

of roughly uniform solid fraction. Similar behavior was observed experimentally by Ridgway and Rupp [15].

The Type *A* wall condition, used for all of the preceding results, yielded substantial slip at the wall in accord with the experimental observations for smooth walls (see Section 5). Therefore a few simulations were subsequently performed with the rather artificial Type *B* wall condition since it had been found to yield no-slip (Campbell and Brennen [5]) and such simulations would allow comparison with the experimental results for roughened walls. The results of one such simulation (α = 30 deg, ϵ_p = 0.6, ϵ_w = 0.8) are presented in Fig. 11. The effect of the change in boundary condition is substantial. Not only does the velocity profile exhibit no slip, but the low solid fraction region next to the wall has disappeared. Also the behavior of the translational temperature close to the wall is rather peculiar; it is low in the first layer of particles due to the no-slip condition but high in the next layer of particles as the high rate of shear of the first layer generates increased translational temperature.

5 Comparison With Experiment

Although the experimental data is limited because of the lack of well-proven instrumentation capable of making local measurements of both solid fraction and velocity, it is valuable to compare the simulation results with the available data. Augenstein and Hogg [1], Bailard [2], Ishida and Shirai [9], Savage [16], and Ridgway and Rupp [15] have all attempted to measure the variation of flow properties with depth in chute flows at different inclination angles.

The form of the velocity profiles obtained from the present simulations are in qualitative agreement with those measured by Augenstein and Hogg [1] and Bailard [2] for both smooth and rough beds, but not with those measured by Savage [16] or Ishida and Shirai [9], who studied rough beds only. The latter measurements contain inflexion points in the profile several particle diameters from the wall, which are not present in the simulations or the other experimental data. The reason for this discrepancy is not clear at present, although Savage [17] suggests that they may represent at least two different types of flow.

A comparison between the velocity profiles of Augenstein and Hogg [1] and those from both Type *A* and Type *B* simulations of flows of similar depth is presented in Fig. 9. These α = 40 deg simulations used ϵ_p = 0.6, ϵ_w = 0.8, values that seemed roughly appropriate for the materials used in the experiments. Augenstein and Hogg performed experiments with various sizes of particles glued to the chute bottom. When the glued particles were the same size as those of the flow the velocity profile exhibited zero slip at the wall. On the other hand, for glued particles smaller than those of the flow a slip increased toward that measured for a smooth surface. Both the magnitude and shape of the simulation velocity profiles agree with the experiments. The Type *A* wall condition yields a profile that appears to correspond to a slightly roughened surface. On the other hand, the Type *B* wall condition yields a profile that corresponds to a fully rough surface. It is clear that the frictional boundary condition in the simulation could be modified so as to produce a family of profiles similar to those of Augenstein and Hogg. Experimental solid fraction profiles for the flow of various sands have been obtained by Bailard [2]. These agree qualitatively with the solid fraction profiles of the simulations. Specifically, Bailard's solid fraction profiles exhibit a layer of reduced ν near the chute bottom and a saltated layer of smaller ν on top of the flow.

Ridgway and Rupp [15] also present solid fraction profiles. However, their profiles are derived from mass flow rate profiles assuming uniform velocity, an assumption that can now be seen to be inappropriate. Consequently, we have reconstructed their mass flow-rate profiles for comparison with the simulation results. But it is also clear that the present two-dimensional flows of cylinders will exhibit higher solid fraction values than a flow of three-dimensional particles. One way of establishing an effective equivalent three-dimensional solid fraction (ν_{3D}) corresponding to a particular simulation value, ν, is to base it on similar interparticle spacing so that $\nu_{3D} = 4/3\,(\nu^3/\pi)^{1/2}$. One example of the corresponding effective three-dimensional mass flow-rate profile was calculated from a simulation of one of the 40 deg chute flows investigated by Ridgway and Rupp [15] and is presented in Fig. 10. The nondimensional mass flow rate, m_{3D} is defined as $\nu_{3D}\langle u\rangle/(gR)^{1/2}$. The agreement in Fig. 10 is as good as could be expected.

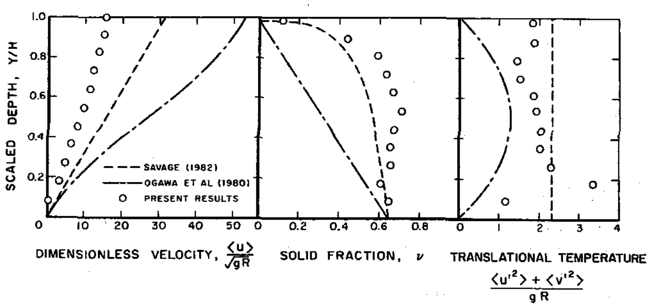

Fig. 11 Comparison of the profiles for a Type *B* simulation (α = 30 deg, ϵ_p = 0.6, ϵ_w = 0.8) with the theoretical results of Savage (1982) and Ogawa et al. (1980)

6 Comparison With Theoretical Profiles

Syntheses of granular material chute flows have been performed by Savage [19] and Ogawa et al. [14]. Both theories represent a significant advance because they attempt to account for temperature. The theory of Ogawa et al. [14] must be considered incomplete as they assumed a linear variation of the solid fraction profile. Both theories require a boundary condition on the mean velocity so a condition of no slip at the wall is applied. Consequently, we will compare their results with those of Type *B* simulation. However, since the theories are three-dimensional, it is necessary to adjust the solid fraction in the same manner as described for Fig. 10; the abscissas of Fig. 11 are therefore $\pi^{1/3}(3\nu_{3D}/4)^{2/3}$. Furthermore, to convert the translational temperature equipartition of energy is assumed so that the temperature abscissas of Fig. 11 are two-thirds the values of the theories. With these crude adjustments the resulting comparison of the simulation and the theoretical models can only be considered qualitative.

The most obvious discrepancy occurs in the velocities. The theories both yielded velocities more than twice the simulation velocities over the bulk of the flow. In this fundamental respect, the simulation results are much closer to the experimental values. The theories yield almost uniform shear rate whereas the shear rate declines with distance from the wall in the simulation. Despite the discrepancy in velocities, Savage's theory is remarkably successful in predicting both the solid fraction profile and the almost uniform temperature. The theory of Ogawa et al. is less successful in both respects.

Surprisingly, the low-density/high-shear rate zone that appears near the wall in the Type *A* simulations are absent from the Type *B* simulation, indicating that the behavior of the flow near solid boundaries is strongly influenced by the wall boundary condition. Heuristically, this effect may be attributed to the large rotational impulse that is applied to a particle on collision with a Type *A* wall. (The average rotation rate, $\langle\omega\rangle$, near the wall of a Type *A* flow is roughly one-half of the slip velocity. In a Type *B* flow, as a particle's rotation is unaffected by collisions with the wall, the average rotation rate $\langle\omega\rangle$ is about one-half of the local shear rate, $d\langle u\rangle/dy$, and consequently, much smaller than found in Type *A* flows.) On subsequent collisions with other particles, the rotational motion will be, in part, transferred to random linear motion causing a rise in the local temperature and a decrease in the local solid fraction. (It should be noted that the experiments of Ridgway and Rupp [15] showed similar behavior for angular sands that would resist rolling on collision with a flat surface.) The choice of appropriate wall boundary conditions for future simulations and theoretical anlayses of granular flow, must be made with great care.

Conclusions

The two-dimensional simulation of the flows of granular materials down inclined chutes described in this paper produce results that agree well with existing experimental data. Both the magnitudes and the profiles of the velocity and the solid fraction agree qualitatively with those of Augenstein and Hogg [1] and Ridgway and Rupp [15].

The simulations using the Type *A* or "smooth" wall boundary condition exhibit a velocity profile with substantial slip at the wall and a solid fraction profile with a layer of lower density next to the wall where the particles have the greatest random motion or temperature. Parenthetically it might also be remarked that the "fluidizing" effect of the low-density layer of highly agitated particles may provide an alternative explanation of certain landslide and avalanche behavior. Such a layer did not occur with a few simulations incorporating a different wall boundary condition (Type *B*) intended to simulate a rough wall by eliminating the slip. The differences generated by this change in the boundary condition emphasize the uncertainty in the detailed mechanics of the particle/wall interactions and the care that must be taken to treat this properly in any theoretical model.

The general consistency with experiment does confirm the validity of the simulations. Consequently, the model can be used to assess flow properties such as the granular temperature, which has been shown to be related to the solid fraction and pressure through an equation of state. It can also be used to provide statistical information on the particle/particle and particle/wall collisions, which is important in improving a basic understanding of granular media flows. (The comparison of the results of the two-dimensional model with actual three-dimensional flows is somewhat questionable. Certain quantities, like the solid fraction, ν, have different interpretations in two and three-dimensions. Also, Evan's [24] recent study showed that two-dimensional fluids do not possess the same transport properties as their three-dimensional counterparts.)

Several phenomena observed in the simulation emphasize the need for further understanding of the production and conduction of granular temperature. Particularly at higher inclination angles the chute flow appears to ride on a less dense layer of highly agitated particles. Artificial vibration of the wall would probably enhance this effect and generate faster flows. (In fact this is a common method of increasing the flow rate from hoppers.)

On the other end of the spectrum, the simulations of the slower flows at α = 20 deg demonstrate how an unshearable plug can develop in the center of the flow. Sufficient internal dissipation must occur within this plug so that the granular heat conducted in from the surrounding shearing flow does not result in a plug temperature sufficient to cause dilation to shearable solid fractions. These observations emphasize the need for the development of a production/dissipation equation for granular temperature similar to that used by Savage [19] and Ogawa et al. [14] as well as the coupling of the temperature to the density, via an equation of state.

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