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Computer simulation of chute flows of granular materials

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1 INTRODUCTION

The purpose of the present paper is to present results from computer simulations of the flow of granular materials down inclined chutes or channels and to compare the results of these calculations with existing experimental measurements of velocity, solid fraction and mass flow rate profiles.

In recent years there has been a significant increase of interest in the detailed mechanics of granular material flows and considerable progress had been made toward understanding the different regimes of flow. The present state of knowledge has recently been reviewed by Spencer (1981) and Savage (1982). The latter comprehensively reviews the recent experimental and theoretical work in higher speed granular flows which lie in the so-called "grain inertia" regime. It is clear that the experimental work in this area is hindered by lack of well-proven, non-intrusive instrumentation necessary to document precisely the velocity, solid fraction and granular temperature distributions within such flows. On the other hand most of the existing theoretical work is limited to modest solid fractions of the order of 0.3 to 0.4. Considerable work is required to extend these theories to solids fractions approaching the critical value. An alternative approach is to utilize the kind of computer simulations which have been quite successful in the analogous problems in molecular gas dynamics (Barker and Henderson (1976), Wood (1975)).

The purpose of such computer simulations is threefold. First they can provide insights and detailed results whcih can be used to suggest appropriate assumptions and to validate theoretical models particularly at higher void fractions. At the present time this is best accomplished using simple shear flows or Couette flows. Simulations of such flows by the present methods are reported in other papers (Campbell (1982), Campbell and Brennen (1982)) where the variation of velocity distributions (deviations from Maxwellian distributions at higher ν), collision angle distributions and other statistical information is provided.

A second objective of the computer simulations is to compare the results with experimental measurements in order to establish appropriate models of the mechanics of particle/particle and particle/wall interactions. Most of the existing measurements of this kind on shear flows come from experiments on granular material flow down inclined (or vertical) channels or "chutes" (see references of Section 5). Thus the objective of this paper is to briefly report on some comparisons between the results of a computer simulation and the existing data for flows down inclined chutes. Further details can be found in Campbell (1982).

Parenthetically one might add, that a third use of computer simulations might be to explore the effects of additional forces such as those due to the interstitial fluid and electrostatic effects. For simplicity, such effects are not included at the present stage of the development of the simulation. Discussion of interstitial fluid effects can be found in other recent works (Savage (1982)).

Other computer models of granular material flows include the work of Cundall (1974), Davis and Deresiewicz (1977), Cundall and Strack (1979), Trollope and Berman (1980) and Walton (1980). Most of these are directed toward the simulation of slower flows and smaller deformations. However, in his original work Cundall (1974) did extend his methods to some higher speed but transient flows such as rockfalls and

and the emptying of a hopper. Also the versatility of Walton's (1980) computer program is readily apparent from the excellent movies which the Lawrence Livermore Laboratory has produced. However to our knowledge none of the existing models have been used to produce continuous "steady" flows which could be used for basic rheological purposes. The intent of the present model was to minimize the complexity of the geometry and the interactions so that steady flows of sufficiently long duration for rheological analysis would be produced. Thus the simulations are just two-dimensional and the particles are circular cylinders. Though extension of the simulation to the third dimension is possible, the necessary computer time would be considerably greater.

2 COMPUTER SIMULATION

The present method for the computer simulation of granular material flows has been documented elsewhere (Campbell (1982), Campbell and Brennen (1982) and will only briefly be described in the present paper. Two dimensional unidirectional flows of inelastic circular cylinders are followed mechanistically. The flow solutions sought have no gradients in the flow direction (x-direction, Figure 1). Though both Couette flows and chute flows have been examined the present paper is confined to discussion of the simulations of chute flow.

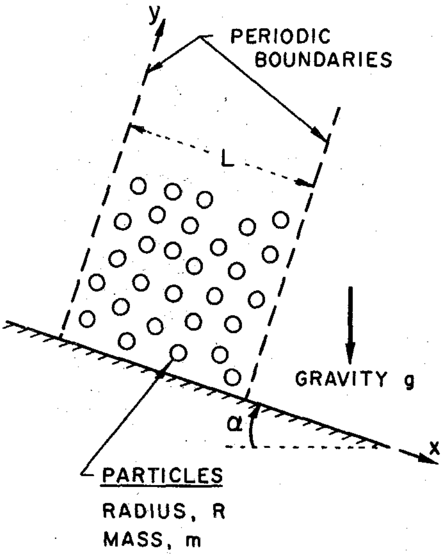

Fig. 1. Schematic of chute flow simulation

Thus the simulation is initiated by placing a number of cylindrical particles in a control volume bounded by the chute bottom and two perpendicular "periodic" boundaries (see Figure 1). A particle passing out of the control volume through one of the periodic boundaries immediately re-enters the control volume through the other periodic boundary at the same relative height above the chute bottom and the same instantaneous velocity. Thus the number of particles initially placed within the control volume remains constant and eventually determines the depth of the flow. The separation, L, between the periodic boundary was adjusted until further doubling of this length (as well as the number of particles) had little effect on the results. For the present results values of L/R (R is the particle radius) of about 10 were found to be sufficient for this purpose (see Figure 2).

All simulations were begun with a randomly perturbed square lattice area of particles in the control volume and randomly chosen instantaneous translation velocities (u,v in the x,y directions of Figure 1) and rotational velocity (ω). Consequently it is convenient to refer to simulations by the dimensions of this initial array; for example, 10 × 4 is a simulation with 10 rows of 4 particles. The intent is to mechanistically follow the entire assemblage of particles, under the action of gravity (g), as it proceeds from collision to collision until a final asymptotic state is reached in which the flow is steady in the sense of being invariant over long time scales. Details of the convergence toward such a state are described in Campbell (1982) and will not be repeated here. The results presented here were run for as long as 200,000 collisions. Though the Couette flows described elswhere required only about 20,000 collisions for convergence of a typical 10 × 4 simulation the present chute flows required considerably longer. Furthermore determination of convergence was clouded by the fact that most of the simulations exhibited the continuous fluctuations typical of small thermodynamic systems (Landau and Lifshitz (1958)). In addition this considerably increased the duration of "steady flow" necessary to perform meaningful evaluation of the flow properties.

In summary, we believe the simulations with chute inclinations of 30° are well converged. However those at $\alpha = 20°$ and $\alpha = 40°$ may still have contained some small deceleration and acceleration respectively at the conclusion of the simulations.

Each individual particle/particle or particle wall collision is assumed to be instantaneous and the resulting departure velocities calculated by conventional means. The two closure conditions used were as follows. First the relative approach and departure velocity components parallel to the line of centers are related by a coefficient of restitution to represent the inelasticity of the collisions. Different coefficients (denoted respectively by ε_p and ε_w) can be used for particle/particle and particle/wall collisions. Values of 0.6 and 0.8 were deemed appropriate as a result of experiments with typical glass beads and aluminum walls and many of the simulations use these values. Secondly a frictional closure condition pertaining to the relative tangential velocities of the contact points upon departure is necessary. For reasons discussed elsewhere (Campbell (1982)) the present simulations were run with a condition requiring zero relative tangential velocity of the contact points upon departure from a collision. This choice is somewhat heuristic. Results using alternative simple choices for this final closure condition for particle/wall collisions are reported elsewhere (Campbell (1982)) and it will be seen in section 5 that this choice can have substantial effect upon the velocity profile.

3 PRESENTATION OF RESULTS

All flow properties were obtained as functions of distance, y, from the chute bottom by dividing the control volume into strips and computing the particle weighted mean of that property in each strip (Campbell (1982)) over a substantial length of time (up to 50,000 collisions). The mean velocity parallel with the chute, $<u>$, and mean solid fraction, ν, were obtained in this way. In addition, data is presented here for the translational temperature (Savage (1982)) defined as the kinetic energy associated with the random translational motions of the particles (divided by $m/2$) as opposed to the kinetic energy associated with the mean shear motion. Specifically the non-dimensional translational temperature is defined as $(<u^2> + <v^2> - <u>^2)/gR$ and this is denoted in the figures by $(<u'^2> + <v'^2>)/gR$. This quantity is of particular interest from a rheological point of view (Savage (1982)) for it is presently believed that the ratio of the typical velocity difference due to shear, $2Rd<u>/dy$, to the typical random translational velocity, $(<u'^2> + <v'^2>)^{\frac{1}{2}}$, plays a central role in determining the constitutive behavior of the flowing granular media (Savage (1982), Jenkins and Savage (1982)).

The results presented here are all nondimensionalized using the intrinsic scales for length, R, for velocity, $(gR)^{\frac{1}{2}}$, and for mass, m (mass of a particle). It should also be noted that the only nongeometric parameters inputed to the simulation are the coefficients of restitution, ε_p and ε_w, and the ratio, β, of the radius gyration of the particles to their radius. All the present simulations were run with $\beta = \frac{1}{2}$, the value appropriate to cylinders.

4 RESULTS OF CHUTE FLOW SIMULATIONS

Three typical instantaneous snapshots of the arrangement of particles in the control volume are presented in Figure 2 for flows at $\alpha = 20°$, $30°$ and $40°$ respectively. Note in particular that as the angle and therefore the velocity and shear rate of the flow is increased the bulk of the flow appears to ride on a lower solid fraction layer of highly agitated particles.

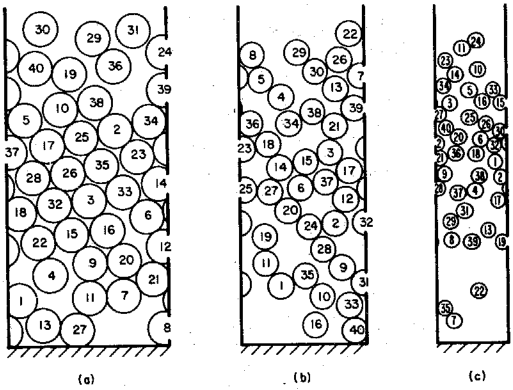

Fig. 2. Typical shapshots from the chute flow simulations for inclinations of (a)$\alpha = 20°$ (b)$\alpha = 30°$ (c)$\alpha = 40°$

Typical velocity, solid fraction and temperature profiles are presented in Figure 3. The velocity profile is much as one would expect and exhibits a slip at the wall which is about half of the velocity of the upper surface. The solid fraction profile exhibits the reduced density layer near the wall which is clearly associated with a higher shear rate and temperature. Parenthetically it might be remarked that the demonstration of the "fluidizing" effect of a low density layer of highly agitated particles near the wall may provide an alternate explanation of the "fluidization" of some of landslides and avalanches.

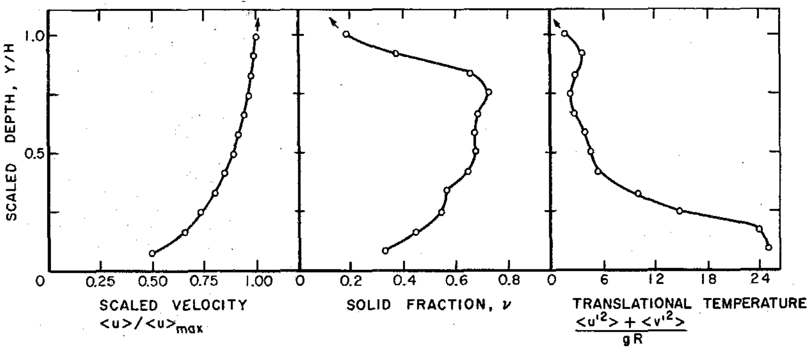

Fig. 3. Typical velocity, solid fraction and translational temperature profiles for a 10×4 simulation with $\alpha = 30°$, $\varepsilon_p = 0.6$, $\varepsilon_w = 0.8$

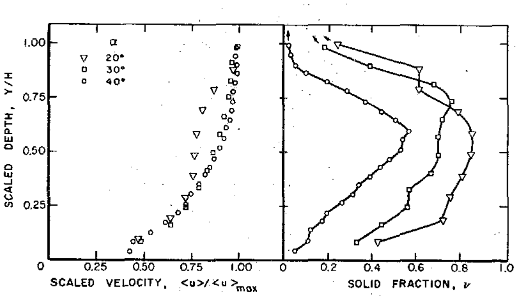

Fig. 4. Velocity and solid fraction profiles for different chute inclinations ($\alpha = 20°$, $30°$ and $40°$) for 10×4 simulations with $\varepsilon_p = 0.6$, $\varepsilon_w = 0.8$.

Typical velocity and solid fraction profiles for each of the inclination angles are included in Figure 4. Note that the decrease in solid fraction near the free surface is a manifestation of a saltated layer of particles on the top of the bulk of the flow.

Note also that the velocity profile for $\alpha = 20°$ deviates from that for $\alpha = 30°$ or $40°$. This is due to the existence of a non-shearing plug in the center of this dense, slow moving flow, a phenomenon that did not occur at higher inclination angles. The data for $\alpha = 20°$ is repeated in Figure 5 in order to further document this interesting example of the coexistence of a high density unshearing plug and a lower density shearing flow. Also indicated in Figure 5 is the square packing solid fraction, ν_s, and the maximum solid fraction which permits the shearing of cylinders, ν_m (Campbell (1982)). The solids fraction exceeds both these values in a central part of the flow and therefore the shear rate, d<u>/dy must be zero at least when $\nu > \nu_m$. However the granular temperature of this plug is not zero. But in the absence of shear this temperature cannot be internally generated. Hence the temperature profile of Figure 5 represents the conduction of fluctuating kinetic energy into the plug from the sheared regions both on top of and below the plug and the subsequent dissipation of this fluctuating kinetic energy by collisions within the matrix of the plug.

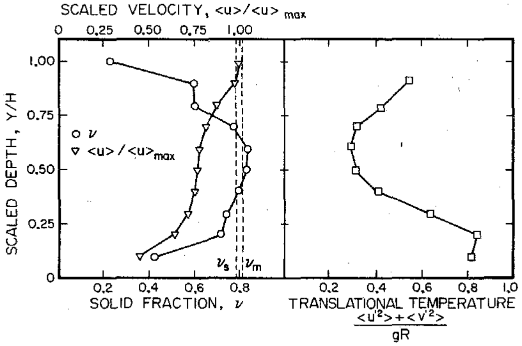

Fig. 5. Velocity, solid fraction and translational temperature profiles for the $\alpha = 20°$ simulation (10×4, $\varepsilon_p = 0.6$, $\varepsilon_w = 0.8$) showing plug flow region.

Typical data for different particle/particle coefficients of restitution, ε_p, is shown in Figure 6. Lower values of ε_p tend to cause blunter, more plug like velocity profiles with higher values of the solid fraction. The three simulations in Figure 6 all have the same number of particles in identical control volumes (10 × 4 simulations). Consequently the higher value of ε_p, the more dilated and deeper the flow. Other simulations with different values of the particle/wall coefficient of restitution (ε_W) showed that this parameter had only a minor effect on the flow (Campbell (1982)).

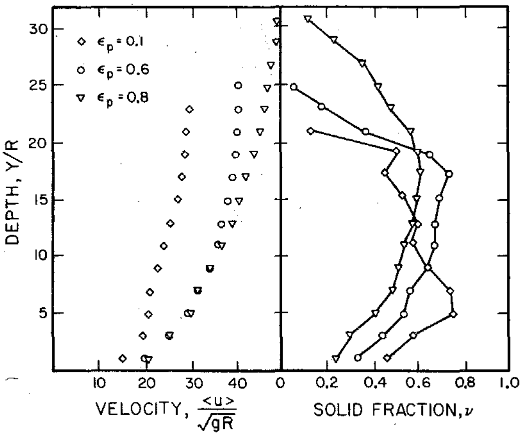

Fig. 6. Effect of particle/particle coefficient of restitution, ε_p, on velocity and solid fraction profiles ($\alpha = 30°$, 10 × 4 simulation, $\varepsilon_W = 0.8$)

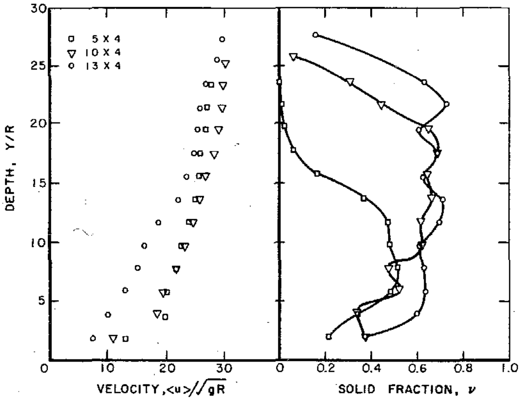

Fig. 7. Effect of flow depth on the velocity and solid fraction profiles of the $\alpha = 30°$ simulation

The effect of the flow depth is demonstrated in Figure 7 using simulations with initial square lattice arrangements of 5 × 4, 10 × 4 and 13 × 4 particles and control volumes of the same width. The velocity variations across the depth tend to increase with increasing depth. Also the maximum solid fraction seems to approach a limiting value which is first attained here in the 10 × 4 simulation; further increase in depth leads to an increase in the extent of this central region of roughly uniform solid fraction. Similar behavior was observed experimentally by Ridgway and Rupp (1970).

5 COMPARISON WITH EXPERIMENT

Though the experimental data base is limited because of the lack of well proven instrumentation capable of making local measurements of both solid fraction and velocity, it is valuable to compare the simulation results with the available data. Augenstein and Hogg (1978), Bailard (1978), Ishida and Shirai (1970), Savage (1979) and Ridgway and Rupp (1970) have all attempted to measure the variation of flow properties with depth in chute flows at different inclination angles.

The form of the velocity profiles obtained from the present simulation are in qualitative agreement with those measured by Augenstein and Hogg (1978) and Bailard (1978) but not with those measured by Savage (1979) or Ishida and Shirai (1979). The latter measurements contain inflexion points in the profile several particle diameters from the wall which are not present in the simulation or the other experimental data. The reason for this discrepancy is not clear at present though Savage (1982) suggests that they may represent at least two different types of flow.

A comparison between the velocity profiles of Augensten and Hogg (1978) and the simulation is presented in Figure 8 for flows with similar depths and a chute inclination of 40°. It is clear that the magnitude of the velocities are in good agreement for the reasonable values of $\varepsilon_p = 0.6$, $\varepsilon_W = 0.8$. More detailed comparison is complicated primarily by questions concerning the appropriate boundary condition at the chute bottom (as well as other lesser complications such as the assessment of sidewall effects). Augenstein and Hogg performed measurements with various sizes of particles glued to the chute bottom to simulate various boundary conditions. Some of these are shown in Figure 8. When the glued particles were larger than those of the flow the velocity profile exhibited greater shear and zero slip at the wall. On the other hand substantial slip occurred for roughnesses smaller than the flowing particles.

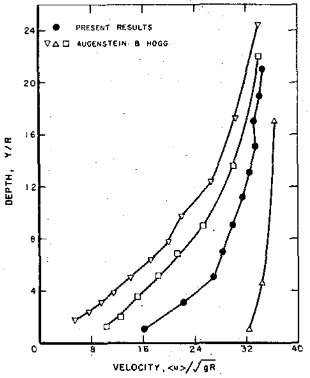

Fig. 8. Comparison of a simulation velocity profile (●, $\alpha = 40°$, 5 × 4 simulation, $\varepsilon_p = 0.6$, $\varepsilon_w = 0.8$) with profiles measured by Augenstein and Hogg; **△**, 35 × 48 mesh sand on smooth stainless steel surface; **▽**, 35 × 48 mesh sand on surface roughened with 35 × 48 mesh sand; **□**, 28 × 35 mesh sand on surface roughened with 65 × 100 mesh sand

The choice of the final closure condition for particle/wall collisions in the simulation (see Section 2) implies some frictional interaction at the boundary. The comparison of Figure 8 suggested that this corresponds to slightly rough walls. It is clear, however, that the simulated boundary condition could be varied in a way that would produce a family of profiles similar to those of Augenstein and Hogg.

Solid fraction profiles for the flow of various sands have been obtained by Bailard (1978). Though no direct comparison is presented in this paper for reasons of space, there is considerable qualitative agreement between Bailard's measured profiles and those of figures 3,4,6 and 7. Both exhibit lower solids fractions near the bottom and in the saltated layer with a central region of higher density. Ridgway and Rupp (1970) also purport to present solid fraction profiles. However their profiles are derived from mass flow rate profiles assuming uniform velocity, an assumption which is questionable. Consequently it is preferable to reconstruct their mass flow rate profiles and compare with the mass flow rate profiles of the simulation. Even then it is clear that the present two-dimemsional flows of cylinders will exhibit higher solid fraction values than a flow of three dimensional particles. One way of establishing an effective equivalent three dimensional solid fraction (ν_{3D}) corresponding to a particular simulation value, ν, is to base it on similar interparticle spacing so that $\nu_{3D} = \frac{4}{3}\left(\frac{\nu^3}{\pi}\right)^{\frac{1}{2}}$. One example of the corresponding effective three-dimensional mass flow rate profile was calculated from a simulation of one of the 40° chute flows investigated by Ridgway and Rupp (1970) and is presented in Figure 9. The non-dimensional mass flow rate, $\dot{m}_{3-D}$ is defined as $\nu_{3D}\langle u\rangle/(gR)^{\frac{1}{2}}$. The agreement in Figure 9 is as good as could be expected.

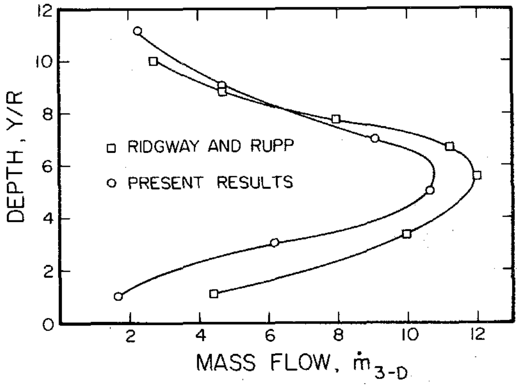

Fig. 9. Comparison of one of the mass flow rate ($\nu_{3D}\langle u\rangle/(gR)^{\frac{1}{2}}$) profiles of Ridgway and Rupp with a simulation of the same flow ($\alpha = 40°$, 4 × 4 simulation).

6 CONCLUSIONS

The two-dimensional simulation of the flows of granular materials down inclined chutes described in this paper produce results which agree well with existing experimental data. Both the magnitudes and the profiles of the velocity, solid fraction and mass flow rates are consistent with the data of Augenstein and Hogg (1978) and Ridgway and Rupp (1970). It is however clear that some work is required to properly correlate the roughness of the solid chute bottom with different models of the particle/wall interaction. However the general consistency with experiment does confirm the validity of the model. Consequently the model can be used with some confidence to asses flow properties such as the granular temperature as well as other statistical information in order to provide detailed understanding of these granular material flows.

Several phenomena observed in the simulation emphasize the need for further understanding of the production and conduction of granular temperature. Particularly at higher inclination angles the chute flow appears to ride on a layer of less dense, highly agitated layer of particles with a high granular temperature. Artificial vibration of the wall could perhaps enhance this effect and accelerate the flow.

On the other hand, the simulations of the slower flows at $\alpha = 20°$ demonstrate how an unshearable plug or matrix can develop in the center of the flow. Sufficient internal dissipation must occur within this plug so that the granular heat conducted in from the surrounding shearing flow does not result in a plug temperature necessary for dilation to shearable solid fractions. These observations emphasize the need for the development of a production/dissipation equation for granular temperature.

7 ACKNOWLEDGEMENTS

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