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Vertical oscillation of a bed of granular material

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ABSTRACT: A bed of granular material which is subjected to vertical vibration will exhibit at least one sudden expansion at a critical acceleration amplitude. This sudden expansion corresponds to a bifurcation similar to that exhibited by a single ball bouncing on a vibrating plate. Theoretical analysis based on this model yields results which are in accord with the experimental observations. Other bifurcations may occur at higher vibration levels.

1 INTRODUCTION

The vibration of granular materials is of interest for a number of reasons. First, vibration is sometimes used instead of an upward flow of gas to fluidize a particle bed reactor and in such devices it is clearly important to know the state of the bed. Secondly, vibration is often used to induce flow in recalcitrant bulk flow transport devices such as hoppers and chutes. It is also used to induce segregation of different density and different size particles. Clearly knowledge of how vibration affects these granular materials provides important design information. As a third incentive, we note that there has been a growing recognition of and interest in the granular state. In a recent review, Jaeger and Nagel (1992) have summarized some of the important issues, questions and applications of knowledge of the granular state and highlight the need for understanding the response to vibration.

Several investigators have previously examined the response of a bed of particles subjected to vertical vibrations (see references) and identified a number of states and transitions between those states. Most investigators agree that within the range of frequencies usually explored (5 $\rightarrow$ 100 Hz) the phenomena are relatively independent of frequency but depend strongly on the acceleration level, $a\Omega^2/g$, and the bed thickness, h_0/d. In this paper we describe the phenomena which were observed to occur as the vertical acceleration of a bed of material is increased and identify a transition or bifurcation similar to that which occurs with a single bouncing ball on a vertically vibrating plate (Wood and Byrne, 1981 and Holmes, 1982).

2 EXPERIMENTS

Experiments were carried out to investigate the behavior of a bed of granular material subjected to vertical vibration. The materials used were A-285 glass beads with a mean diameter of 0.292 cm. Various quantities of these beads were placed in a rectangular box with cross-section dimensions of 11 cm × 13.2 cm which was in turn mounted on an electro-mechanical shaker and subjected to vertical vibration at frequencies between 4 and 10 Hz with amplitudes up to about 2.5 g. An accelerometer was used to measure the acceleration level accurately.

The box had a thick aluminum base and back but the other three sides were made of lucite so that the behavior of the beads could be observed. Paper lids of various thickness were placed on top of the beads leaving a clearance of about 1 mm between the edge of the lid and the walls of the box. When the box was vibrated vertically, the bed of beads would expand and the lid would float on the beads. Fortunately, the lid proved to be quite stable and would remain horizontal and centralized with roughly equal spacing all around the periphery. Because this spacing was smaller than the diameter of the beads, all of the beads would remain under the lid. A stroboscope was used to examine the motion of the lid and the beads during various parts of the oscillation cycle. By this means we were able to observe that the spacing, h, between the base and the lid did not vary greatly during the oscillations. The beads would bounce around below the lid but because of the resistance to the flow of air around the sides of the lid, the volume of beads and air would remain almost constant during a cycle of oscillation. Thus, using the strobe and a scale attached to the exterior of the box, it was possible to measure the height, h, for each operating condition.

Experiments were conducted by observing the evolution of the bed of beads as the vibration amplitude, a, was increased from zero to the maximum of which the shaker was capable. Such experiments were conducted over a range of frequencies (4→10 Hz and Ω will denote the radian frequency) for various quantities of beads and for lids with different masses as follows:

Experiment No.	*Bead Mass (gm.)*	*Lid Mass (gm.)*
1	250	3.44
2	125	3.44
3	125	7.17
4	375	3.44
5	125	17.06
6	625	3.44
7	45	3.51
8	125	28.14

It should be noted that a single packed layer of beads resting on the base of the box would weigh 54 grams. Consequently the masses of beads range from less than a single layer to about eleven layers. The 45 gm. of experiment 7 was close to the minimum at which the lid would remain horizontal for the duration of the experiment.

3 EXPERIMENTAL RESULTS

The results for the base-to-lid spacing, h, as a function of vibration amplitude will be presented in various ways but we focus here on the expansion of the bed, $h^* = h - h_o$, where h_o is the spacing at rest. For reasons which will become clear, h^* will be presented both as a function of the acceleration amplitude $a\Omega^2$ (or rather $a\Omega^2/g$ where g is the acceleration due to gravity) and as a function of the vibration velocity $a\Omega$. The typical behavior of the bed is best illustrated by the results from experiment 7 which are presented in figure 1. The bed would begin to expand at an acceleration amplitude of about 1 g and this expansion would gradually increase until a critical value of the acceleration amplitude, $(a\Omega^2/g)_c$ was reached, which appeared to be independent of frequency but to vary with both the mass of beads and the mass of the lid. At this critical acceleration amplitude the lid would rise quite abruptly and then settle down at a substantially larger spacing, h^*. As illustrated in figure 1, further increase in the acceleration would result in further bed expansion but this was more gradual than the expansion encountered during transition. The top graph in figure 1 illustrates the fact that the critical conditions appear to occur at a given acceleration amplitude regardless of the frequency. On the other hand, the bottom graph in figure 1 illustrates the fact that the supercritical conditions correlate with the velocity amplitude, $a\Omega$, rather than the acceleration amplitude.

Using the strobe, one could observe that prior to the transition, the motions of the particles were fairly uncoordinated. However, above the transition the beads began to move as a block which collided once per cycle with the base and with the lid. The collision with the base seemed quite inelastic and it appeared that the block only left the base again when the acceleration of the base exceeded some critical value. However it is also important to emphasize that the block expands and contracts substantially during each cycle being quite concentrated while it is in contact with the base but quite dilute while it is in flight.

In order to understand the fundamental dynamics behind the above phenomena it is

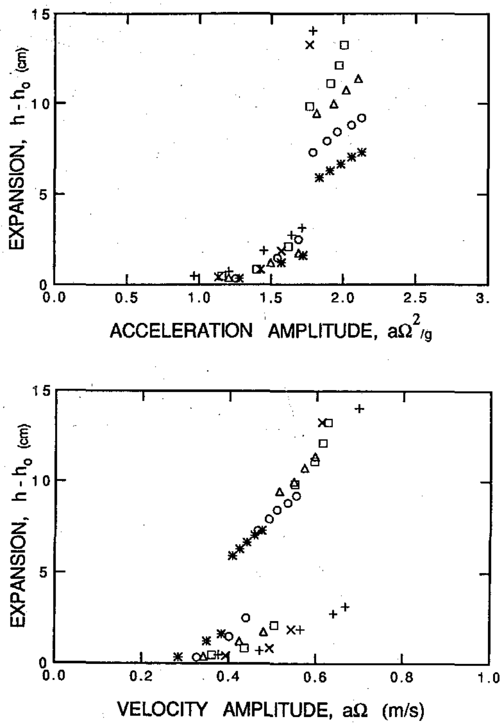

Figure 1. The dependence on the bed expansion, $h\text{-}h_o$, on the acceleration amplitude, $a\Omega^2/g$, and the velocity amplitude, $a\Omega$ (in m/s), for experiment 7. Various frequencies as follows: 4 Hz = +, 4.5 Hz = X, 5 Hz = O, 5.5 Hz = Δ, 6 Hz = □, 7 Hz = *.

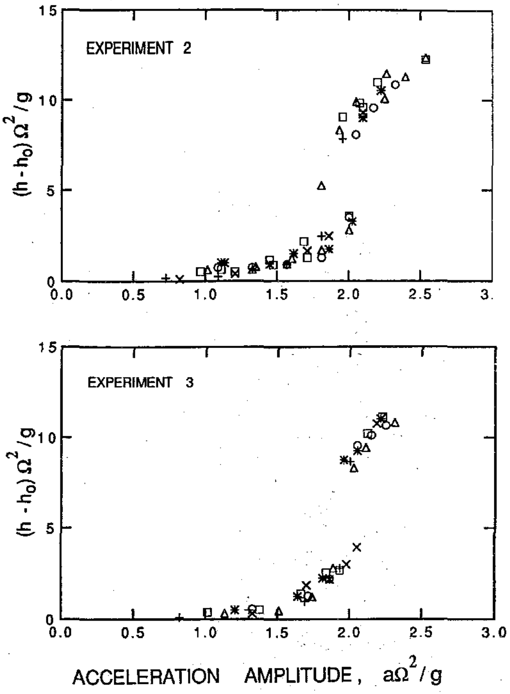

Figure 2. Dimensionless expansion $(h\text{-}h_o)\Omega^2/g$, plotted against the dimensionless acceleration for experiments 2 and 3. Frequency key as in figure 1.

valuable to present the data non-dimensionally. This is accomplished by non-dimensionalizing the expansion as $\Omega^2(h\text{-}h_o)/g$ and plotting this versus the nondimensional acceleration amplitude, $a\Omega^2/g$. Examples from experiments 2 and 3 are shown in figure 2 in which the subcritical and supercritical data clearly form two distinct groups of points. Indeed the two groups of points both appear to lie close to quadratic curves which imply that each group of points correspond to a roughly constant value of the inverse Froude number,

$$Fr^{-1} = \frac{\left(g(h-h_o)\right)^{1/2}}{a\Omega}$$

To examine this further, the inverse Froude number is plotted versus the acceleration, $a\Omega^2/g$, in figure 3 for the typical data of experiments 2 and 3. It seems particularly noteworthy that the subcritical data corresponds roughly to an inverse Froude number, Fr^{-1}, of between 0.5 and 1.0 and that the supercritical corresponds quite closely to $Fr^{-1} = 1.5$ (recall that the values of $(h\text{-}h_o)$ and a for some of the subcritical data are quite small and this may account for the larger scatter in that group of points).

4 THEORETICAL ANALYSIS

The analytical solution to the problem of a ball bouncing on a horizontal flat plate performing vertical oscillations (amplitude, a, and radian frequency, Ω) are of interest for several reasons. First, the model could be considered appropriate for individual particles when particle/particle collisions are relatively rare, as, for example, in the case where less than a single layer of particles was used. Alternatively, in the case of a larger mass of particles, the solution might be

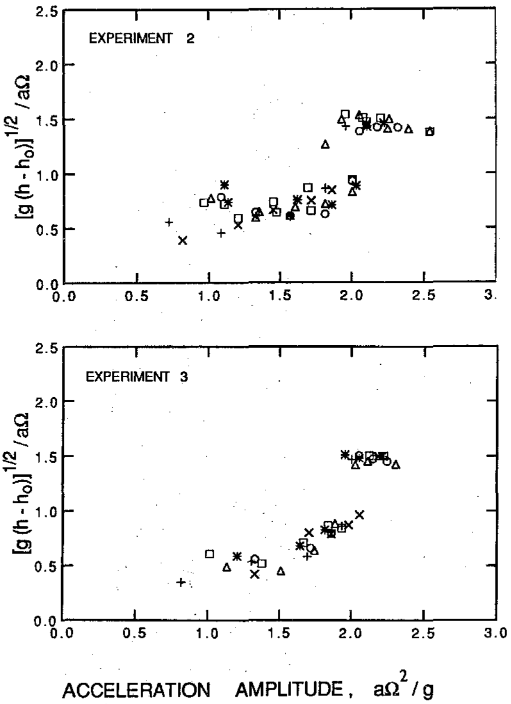

Figure 3. Inverse Froude number, $[g(h-h_o)]^{1/2}/a\Omega$, plotted against dimensionless acceleration for experiments 2 and 3. Frequency key as in figure 1.

considered applicable to the whole mass when it performs a coherent periodic motion. In either case, we shall consider that the particles bounce off a lid which, by some unspecified damping mechanism, is maintained at a constant height above the oscillating plate. The lid is however entirely supported in the mean by the impulses imparted by the particles; thus solutions will be sought for various ratios of the lid mass to the particle mass, f. The problem also requires specification of the coefficients of restitution, ε_P and ε_L, for the collisions with the plate and lid respectively.

The dynamics of the ball bouncing problem without a lid have now become a classic example of the occurrence of bifurcations and we shall see that this seems the probable explanation for the experimentally observed transition.

The first, simple solution which is useful is that for no lid and for ε_P=0. The ball remains in contact with the plate until the latter is accelerating downward at an acceleration equal to g. The maximum height, h_S, to which the ball rises above the plate can readily be identified parametrically as

$$\frac{\Omega^2 h_s}{g} = \frac{a\Omega^2}{g}\left[(x_2 - x_1)\cos x_1 + \sin x_1 - \sin x_2\right] \quad (1)$$

where

$$\sin x_1 = g / a\Omega^2; \; x_2 - x_1 = \frac{a\Omega^2}{g}(\cos x_1 - \cos x_2)$$

This relationship between the dimensionless "expansion," $\Omega^2 h_S/g$ and the acceleration amplitude was obtained numerically and is identified in figure 4 as the "no bounce" solution. Note that it corresponds quite closely with the subcritical experimental data (in figure 4 we have used the data of experiment 2 as typical). When one examines the specifics of this solution for the range of $\Omega^2 a/g$ values of interest here (less than about 2) one finds that after becoming airborne the particle (or particle mass) will return to impact the plate after less than about 0.6 of a cycle. Even if ε_P were non-zero and there were several small bounces following this impact there is more than sufficient time left in the cycle for the particle (or particle mass) to effectively come to rest on the plate before the next occurrence of a downward acceleration of 1 g. Thus the solution is valid for a range of ε_P.

The second benchmark which is of interest is the periodic solution in which the particle (or particle mass) bounces off the plate and off the lid once per cycle of plate oscillation. In order for such a periodic solution to exist, the relative velocity of departure from the lid collision, u_4, the relative velocity of incidence on the plate, u_1 (both u_4 and u_1 considered positive downward), the relative velocity of departure from the plate, u_2, and the relative velocity of incidence on the lid, u_3, (u_2 and u_3 considered positive upward) must be given by

$$\frac{\Omega u_1}{g} = \frac{2\pi(1+f)}{(1+\varepsilon_P)}; \quad \frac{\Omega u_2}{g} = \frac{2\pi\varepsilon_P(1+f)}{(1+\varepsilon_P)} \quad (2)$$

$$\frac{\Omega u_3}{g} = \frac{2\pi f}{(1+\varepsilon_L)}; \quad \frac{\Omega u_4}{g} = \frac{2\pi\varepsilon_L f}{(1+\varepsilon_L)}$$

The solution is most readily obtained parametrically by selecting the times t_1 and t_2 during a cycle when collision with the plate and the lid, respectively, occur. It then follows that

$$\frac{a\Omega^2}{g} = \left[2\pi(\cos\Omega t_1 + \cos\Omega t_2)\right]^{-1} \times \left[\Omega(t_1 - t_2)\left(\frac{\Omega u_2}{g} + \frac{\Omega u_3}{g}\right) + \{\Omega(t_1 - t_2) + 2\pi\}\left(\frac{\Omega u_1}{g} + \frac{\Omega u_4}{g}\right)\right] \qquad (3)$$

and that the expansion, h, defined as the increase in the spacing between the plate and the lid is given by

$$\frac{h}{a} = \sin\Omega t_1 - \sin\Omega t_2 + \frac{\Omega(t_2 - t_1)}{2}\left[\frac{u_2}{a\Omega} + \frac{u_3}{a\Omega} + \cos\Omega t_1 + \cos\Omega t_2\right] \qquad (4)$$

Thus the choice of two arbitrary values of Ωt_1 and Ωt_2 corresponds to a solution for specific values of f and $a\Omega^2/g$ and yields a specific value for h/a. In addition one must check to ensure that there are no unforeseen overlaps between the particle and the lid or plate during the oscillation cycle. Typical results for this analysis are included in figure 4 (identified as the "with bounce" solution) for $\varepsilon_P = 0.25$, $\varepsilon_L = 0$ and $f = 0.01$, 0.1 and 0.2. Note that for a given lid and given coefficients of restitution there exist no periodic solutions of this type for accelerations below a certain critical level. It should be noted in passing that there is a large variety of other possible periodic solutions. For example there exist the possibilities of one bounce for every two or more plate cycles and of two or more bounces in a single plate cycle. Alternatively the ball might cycle through two or more types of bounce before repeating itself.

It is important to point out that studies of the dynamics of the much simpler system of a single particle on a vibrating plate (Wood and Byrne, 1982 and Holmes, 1982) have revealed a system of bifurcations at different critical values

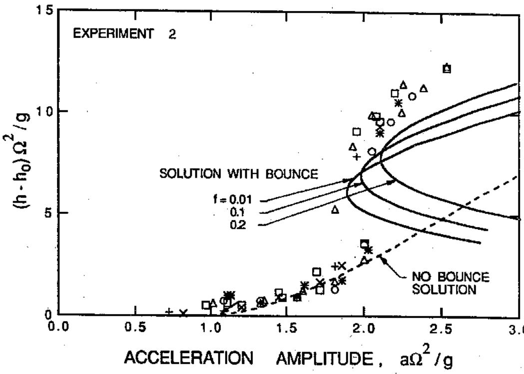

Figure 4. Typical data (from experiment 2) compared with the analytical solutions described in the text.

of the acceleration, $\Omega^2 a/g$. As the acceleration amplitude is increased the first bifurcation occurs at

$$(\Omega^2 a/g)_{CRITICAL} = \frac{\pi(1-\varepsilon_P)}{1+\varepsilon_P} \qquad (5)$$

The experimental data is clearly indicating that such a bifurcation also occurs with the granular mass. Though the analogy may only be of qualitative value, it is nevertheless of interest to observe that equation (5) yields $(\Omega^2 a/g)_{CRITICAL} = 1.88$ when $\varepsilon_P = 0.25$, which is qualitatively consistent with the current experimental data since the effective ε_P for the mass of particles may be as low as 0.25. We have chosen to use $\varepsilon_P = 0.25$ to demonstrate the results of the analytic calculations.

The above analysis is clearly consistent with the following explanation of the observed experimental behavior. At small values of the acceleration just above 1 g, the data is consistent with the simple, no-bounce solution. However when the acceleration approaches the critical or bifurcation value of $\Omega^2 a/g$ a sudden expansion of the bed occurs as the particle mass begins to move as a fairly coherent whole, bouncing off the plate once per plate oscillation cycle.

In addition to the previous analysis, a computer simulation was used to examine the dynamics of a column of inelastic balls bouncing on a sinusoidally vibrating plate. The maximum separation height between the top ball and the plate was recorded for various $a\Omega^2/g$ and ε. For one ball, the simulation results match the experimental data in the subcritical region best when $\varepsilon = 0$. The $a\Omega^2/g|_{CRITICAL}$ and h^* in the supercritical region however is matched best when $\varepsilon = 0.327$. Additional simulations were performed for a column of five balls. Although the simulation results did not quantitatively match the experimental data, the characteristic transition still occurs for this more complex system. It is also of interest to note that several transitions occur similar as in the case of just one ball.

CONCLUSIONS

A bed of granular material which is subjected to vertical vibration will exhibit at least one sudden expansion at a critical acceleration amplitude. This sudden expansion corresponds to a bifurcation similar to that exhibited by a single ball bouncing on a vibrating plate. Theoretical analysis based on this model yields results which are in accord with the experimental observations. Other bifurcations may occur at higher vibration levels.

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