3.9.1 Unsteady one-speed diffusion theory

To exemplify the nature of nuclear reactor kinetics it is convenient to return to the basic one-speed diffusion equation 1, section 3.6.2. Retaining the unsteady term, $\partial\phi/\partial t$, this becomes

$$\frac{1}{\bar{u}}\frac{\partial\phi}{\partial t} - D\bigtriangledown^2\ \phi = S - \Sigma_a\phi \tag{1}$$

where it is assumed that the diffusivity is uniform throughout the reactor and does not change with time. Recall that equation 1 is a statement of neutron conservation in some small piece of the core in which the excess of the neutrons produced over the neutrons absorbed (the right hand side) is balanced by the rate of increase of the neutrons in that piece plus the net flux of neutrons out of that piece of core (the left hand side). As before the left hand side is set equal to $(k_\infty - 1)\Sigma_a\phi$ so that the diffusion equation 1 becomes

$$\frac{1}{\bar{u}D}\frac{\partial\phi}{\partial t} - \bigtriangledown^2\ \phi = \frac{(k_\infty - 1)\Sigma_a}{D}\ \phi \tag{2}$$

Fortunately, equation 2 is linear in the neutron flux, ϕ, and therefore solutions are superposable. Consequently, for simplicity, the focus will be on a single basic solution knowing that more complex solutions may be constructed by superposition. This basic solution for the neutron flux, $\phi(x_i, t)$, takes the form:

$$\phi(x_i, t) = C\exp(-\xi t)\phi^*(x_i) \tag{3}$$

where C is a constant, ξ is the time constant associated with the transient and ϕ^* is a time-independent neutron flux function. Substituting from equation 3 into the governing equation 2 yields the following relation for ϕ^*:

$$\bigtriangledown^2\ \phi^* + \left\{\frac{(k_\infty - 1)\Sigma_a}{D} + \frac{\xi}{D\bar{u}}\right\}\phi^* = 0 \tag{4}$$

Note that, as in the steady state case, Σ_a could be replaced using $\Sigma_a = D/L^2$ where L is the neutron diffusion length, L (see the definition 2, section 3.6.2). In parallel with the steady state equation 5, section 3.6.3, equation 4 can be written as the eigenequation

$$\bigtriangledown^2\ \phi^* + B_g^2\phi^* = 0 \tag{5}$$

where the geometric buckling, B_g, is the specific eigenvalue for the particular geometry of the reactor under consideration. Since equation 5 is identical to that governing ϕ in the steady case, and since the boundary conditions are usually the same, the geometric buckling, B_g, will be the same as in the steady case. In addition, from equations 4 and 5 it follows that

$$B_g^2 = \frac{(k_\infty - 1)}{L^2} + \frac{\xi}{D\bar{u}} \tag{6}$$

so that, using equation 2, section 3.6.2,

$$\xi = D\bar{u}B_g^2 + \bar{u}\Sigma_a - k_\infty \bar{u}\Sigma_a \tag{7}$$

in which the left hand side consists of contributions to ξ from the neutron leakage, absorption and production respectively. Alternatively the quantity t^* can be defined by

$$\xi = \frac{(k-1)}{t^*} \tag{8}$$

where k is the multiplication factor and t^* is the mean lifetime of a neutron in the reactor where using equations 10, section 3.6.3, and 2, section 3.6.2,

$$k = \frac{k_\infty}{1 + L^2 B_g^2} \qquad \text{and} \qquad t^* = \frac{1}{\bar{u}\Sigma_a(1 + L^2 B_g^2)} \tag{9}$$

Note that $(\bar{u}\Sigma_a)^{-1}$ is the typical time before absorption and $(\bar{u}\Sigma_a L^2 B_g^2)^{-1}$ is the typical time before escape; combining these it follows that t^* is the typical neutron lifetime in the reactor.

Hence the solution to the characteristic unsteady problem may be written as

$$\phi = C \exp\left(-\frac{(k-1)}{t^*}t\right) \, \phi^* \tag{10}$$

where ϕ^* is the solution to the steady diffusion problem with the same geometry and boundary conditions. The characteristic response time of the reactor, t_R, is known as the *reactor period*. In the absence of other factors, this analysis and equation 10 suggest that t_R might be given by

$$t_R = \frac{t^*}{(k-1)} \tag{11}$$

Since the typical lifetime of a neutron, t^*, in a LWR is of the order of 10^{-4} *sec*, equation 11 suggests that a very small perturbation in the multiplication factor k of 0.1% to 1.001 might result in a reactor period, t_R, of 0.1 *sec* and therefore more than a 2×10^4 fold increase in the neutron population in one second. This would make any reactor essentially impossible to control. Fortunately, as described in section 2.3.4, delayed neutron emission causes a more than 100 fold increase in the mean neutron lifetime in an LWR and a corresponding increase in the reactor period, making reactor control much more manageable (see section 4.3.6).