3.8.3 Control rod lattice cell

The obverse of the fuel rod lattice cell is the control rod lattice cell in which an individual control rod ($0 < r < R_1$) is surrounded by an annulus ($R_1 < r < R_2$) containing a homogeneous mix of fuel rod and coolant as can also be depicted by figure 1.

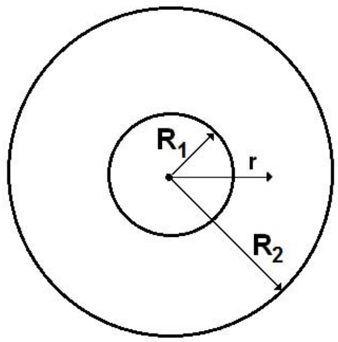

Figure 1: Generic circular lattice cell.

Then the governing equations for the neutron flux are:

$$\bigtriangledown^2 \phi_1 - \frac{\phi_1}{L_1^2} = 0 \qquad \text{in} \qquad r \leq R_1 \tag{1}$$

$$\bigtriangledown^2 \phi_2 + B_g^2 \phi_2 = 0 \qquad \text{in} \qquad R_1 \leq r \leq R_2 \tag{2}$$

where gradients in the direction normal to the sketch (the z direction) are neglected and L_1 is the neutron mean free path in the control rod. The boundary conditions are the same as in the fuel rod lattice cell and it follows that the appropriate general solutions are

$$\phi_1 = C_1 I_0(r/L_1) + C_2 K_0(r/L_1) \tag{3}$$

$$\phi_2 = C_3 J_0(B_g r) + C_4 Y_0(B_g r) \tag{4}$$

Since ϕ_1 must be finite at $r = 0$, C_2 must be zero. If, again for convenience, the diffusivities are assumed to be the same in both regions ($D_1 = D_2 = D$) then the boundary conditions 9 and 10, section 3.7.4 require that

$$C_1 I_0(R_1/L_1) = C_3 J_0(B_g R_1) + C_4 Y_0(B_g R_1) \tag{5}$$

$$C_1 I_1(R_1/L_1)/L_1 = -C_3 B_g J_1(B_g R_1) - C_4 B_g Y_1(B_g R_1) \tag{6}$$

In addition the outer boundary condition 9, section 3.7.4, requires that

$$C_3 J_1(B_g R_2) + C_4 Y_1(B_g R_2) = 0 \tag{7}$$

and equations 5, 6 and 7 lead to the eigenvalue equation

$$I_1(R_1/L_1)\,[J_0(B_gR_1)Y_1(B_gR_2) - J_1(B_gR_2)Y_0(B_gR_1)] =$$
$$B_gL_1I_0(R_1/L_1)\,[J_1(B_gR_2)Y_1(B_gR_1) - J_1(B_gR_1)Y_1(B_gR_2)] \qquad (8)$$

Given the non-dimensional parameters R_2/R_1 and L_1/R_1, the solution to this equation yields the non-dimensional geometric buckling, B_gR_1, for this configuration. When the neutron mean free path, L_1, is large relative to R_1 and R_2 the approximate solution to equation 8 is

$$B_g^2L_1^2 \approx R_1^2/(R_2^2 - R_1^2) \qquad (9)$$

More precise solutions for the non-dimensional geometric buckling, $B_g^2R_1^2$, are shown in figure 2 for various values of L_1/R_1 and R_2/R_1. These lead to different neutron flux profiles as exemplified by those presented in figure 3. As expected the flux inside the control rod ($r < R_1$) is smaller than in the surroundings but the profile flattens out as the geometric buckling decreases.

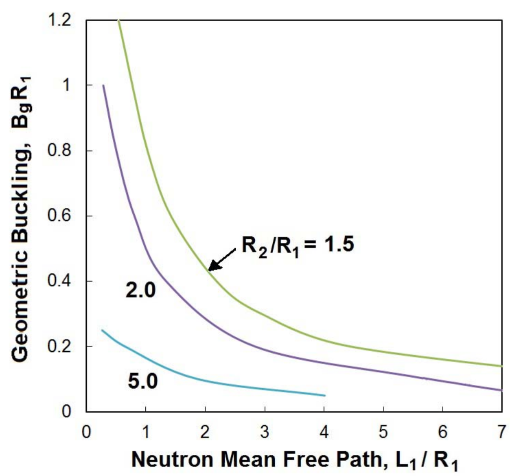

Figure 2: Values of the non-dimensional geometric buckling for the control rod lattice cell as a function of L_1/R_1 for three values of R_2/R_1 as shown.

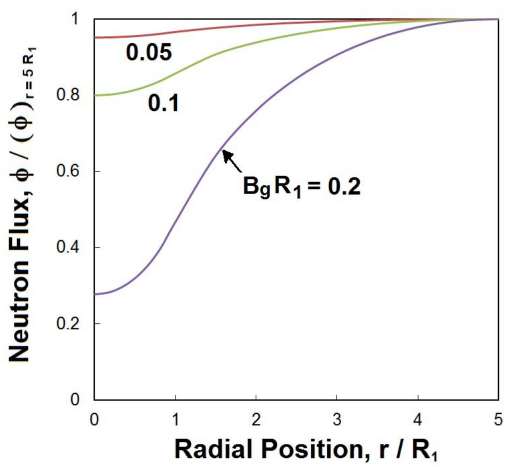

Figure 3: Typical neutron flux profiles for the control rod lattice cell with $R_2/R_1 = 5$ for three values of non-dimensional geometric buckling, B_gR_1, as shown.