3.8.2 Fuel rod lattice cell

The diffusion theory solution for the single fuel rod lattice cell requires the solution of the following forms of the diffusion equation 3, section 3.6.3, for the neutron fluxes ϕ_1 and ϕ_2 in the two regions of the cell sketched in figure 1:

$$\bigtriangledown^2 \phi_1 + B_g^2 \phi_1 = 0 \qquad \text{in} \qquad r \leq R_1 \tag{1}$$

$$\bigtriangledown^2 \phi_2 - \frac{\phi_2}{L_2^2} = 0 \qquad \text{in} \qquad R_1 \leq r \leq R_2 \tag{2}$$

subject to the boundary conditions 1 and 2, section 3.8.1, and neglecting any gradients in the direction normal to the figure 1 sketch (the z direction). The

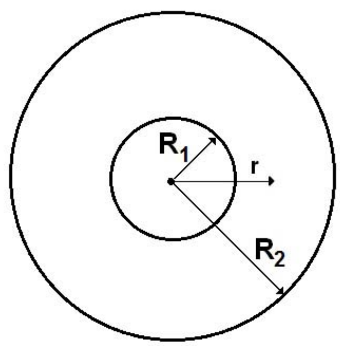

Figure 1: Generic circular lattice cell.

appropriate general solutions are

$$\phi_1 = C_1 J_0(B_g r) + C_2 Y_0(B_g r) \tag{3}$$

$$\phi_2 = C_3 I_0(r/L_2) + C_4 K_0(r/L_2) \tag{4}$$

where $J_0()$ and $Y_0()$ are Bessel functions of the first and second kind, $I_0()$ and $K_0()$ are modified Bessel functions of the first and second kind and C_1, C_2, C_3, and C_4 are constants yet to be determined. Since ϕ_1 must be finite at $r = 0$, C_2 must be zero. If, for the convenience of this example, the diffusivities are assumed to be the same in both regions ($D_1 = D_2 = D$) then the boundary conditions 8, section 3.7.4, require that

$$C_1 J_0(B_g R_1) = C_3 I_0(R_1/L_2) + C_4 K_0(R_1/L_2) \tag{5}$$

$$-C_1 B_g J_1(B_g R_1) = C_3 I_1(R_1/L_2)/L_2 - C_4 K_1(R_1/L_2)/L_2 \tag{6}$$

where $J_1()$, $I_1()$ and $K_1()$ denote Bessel functions of the first order. In addition the outer boundary condition 7, section 3.8.2, requires that

$$-C_3 I_1(R_2/L_2) + C_4 K_1(R_2/L_2) = 0 \tag{7}$$

and equations 5, 6 and 7 lead to the eigenvalue equation

$$J_0(B_gR_1)\,[I_1(R_2/L_2)K_1(R_1/L_2) - I_1(R_1/L_2)K_1(R_2/L_2)] =$$

$$L_2B_gJ_1(B_gR_1)\,[I_0(R_1/L_2)K_1(R_2/L_2) + I_1(R_2/L_2)K_0(R_1/L_2)] \qquad (8)$$

Given the non-dimensional parameters R_2/R_1 and L_2/R_1 the solution to this equation yields the non-dimensional geometric buckling, B_gR_1, for this configuration. When the neutron mean free path, L_2, is large relative to R_1 and R_2 the approximate solution to equation 8 is

$$B_g^2L_2^2 \approx (R_2^2 - R_1^2)/R_1^2 \qquad (9)$$

More precise solutions for the non-dimensional geometric buckling, $B_g^2R_1^2$, are shown in figure 2 for various values of L_2/R_1 and R_2/R_1. These lead to different neutron flux profiles as exemplified by those presented in figure 3. As expected the flux inside the fuel rod is larger than in the surroundings but the profile flattens out as the neutron mean free path increases.

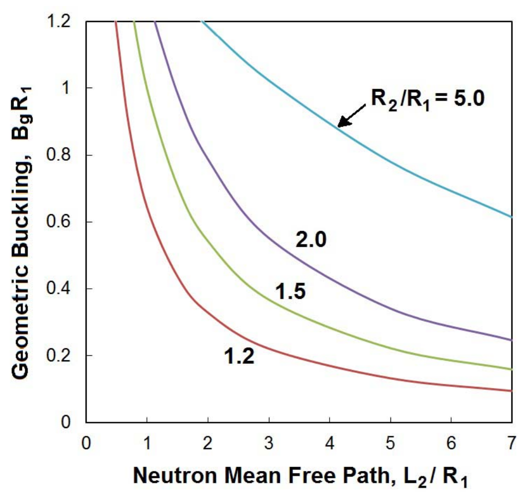

Figure 2: Values of the non-dimensional geometric buckling for the fuel rod lattice cell as a function of L_2/R_1 for four values of R_2/R_1 as shown.

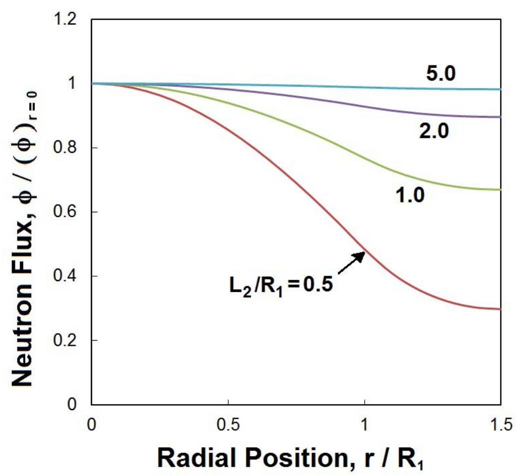

Figure 3: Typical neutron flux profiles for the fuel rod lattice cell with $R_2/R_1 = 1.5$ for four values of L_2/R_1 as shown.