3.7.4 Effect of control rod insertion

A second example of a practical modification of the diffusion theory solutions is to consider a core into which control rods have been partially inserted so that, as sketched in figure 1, the reactor core consists of two regions with different levels of neutron absorption. The fractional insertion will be denoted by β. Assuming that the control rod absorption is sufficiently large so that the conditions in the controlled region are subcritical the equations governing the neutron flux in the two regions are

$$\bigtriangledown^2 \phi_1 + B_g^2 \phi_1 = 0 \qquad \text{in} \qquad 0 \leq z \leq (1-\beta) H_E \tag{1}$$

$$\bigtriangledown^2 \phi_2 - \frac{\phi_2}{L_2^2} = 0 \qquad \text{in} \qquad (1-\beta) H_E \leq z \leq H_E \tag{2}$$

where subscripts 1 and 2 refer to the two regions indicated in figure 1, L_2 is the neutron diffusion length in region 2 and, for convenience, the origin of z has been shifted to the bottom of the core. The boundary conditions on the cylindrical surface $r = R_E$ are $\phi_1 = \phi_2 = 0$ (as in section 3.7.1) and on the radial planes they are

$$\phi_1 = 0 \quad \text{on} \quad z = 0 \quad ; \quad \phi_2 = 0 \quad \text{on} \quad z = H_E \quad ; \tag{3}$$

$$\phi_1 = \phi_2 \quad \text{and} \quad \frac{\partial \phi_1}{\partial z} = \frac{\partial \phi_2}{\partial z} \quad \text{on} \quad z = (1-\beta) H_E \tag{4}$$

where, for simplicity, it has been assumed that the neutron diffusivities are the same in both regions. By separation of variables, the appropriate solutions to

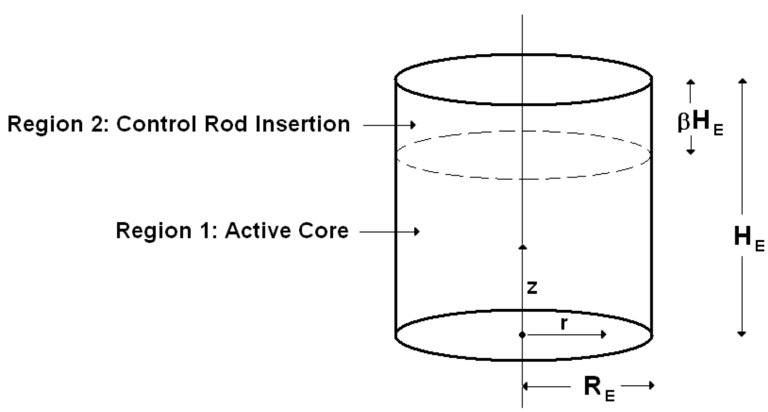

Figure 1: Cylindrical reactor with partial control rod insertion.

equations 1 and 2 are

$$\phi_1 = [C_1 \sin \xi_1 z + C_2 \cos \xi_1 z]\, J_0(2.405 r/R_E) \tag{5}$$

$$\phi_2 = \left[C_3 e^{\xi_2 z} + C_4 e^{-\xi_2 z}\right] J_0(2.405r/R_E) \tag{6}$$

where C_1, C_2, C_3, C_4, $\xi_1$and ξ_2 are constants as yet undetermined and the boundary conditions at $r = R_E$ have already been applied. The governing equations 1 and 2 require that

$$\xi_1^2 = B_g^2 - (2.405/R_E)^2 \quad ; \quad \xi_2^2 = (1/L_2)^2 + (2.405/R_E)^2 \tag{7}$$

The boundary conditions 3 require that

$$C_2 = 0 \quad ; \quad C_4 = -C_3 e^{2\xi_2 H_E} \tag{8}$$

and using these with the boundary conditions 4 yields

$$C_1 \sin\{\xi_1(1-\beta)H_E\} = -C_3 e^{\xi_2 H_E}\left[e^{\xi_2\beta H_E} - e^{-\xi_2\beta H_E}\right] \tag{9}$$

$$\xi_1 C_1 \cos\{\xi_1(1-\beta)H_E\} = \xi_2 C_3 e^{\xi_2 H_E}\left[e^{\xi_2\beta H_E} + e^{-\xi_2\beta H_E}\right] \tag{10}$$

Eliminating the ratio C_1/C_3 from these last two expressions yields

$$\xi_2 \tan\{\xi_1(1-\beta)H_E\} + \xi_1 \tanh\{\xi_2\beta H_E\} = 0 \tag{11}$$

Since ξ_1 and ξ_2 are given by equations 7 this constitutes an expression for the critical size of the reactor, R_E (or R) given the aspect ratio H_E/R_E as well as B_g, L_2 and β. Equivalently it can be seen as the value of β needed to generate a critical reactor given R_E, H_E, B_g and L_2.

As a non-dimensional example, figure 2 presents critical values for the fractional insertion, β, as a function of the quantity $B_g R_E$ (which can be thought of as a non-dimensional size or non-dimensional geometric buckling) for a typical aspect ratio, H_E/R_E, of 2.0 and several values of L_2/R_E. Naturally the critical size increases with the insertion, β; equivalently the insertion, β, for a critical reactor will increase with the *size* given by $B_g R_E$. Note that the results are not very sensitive to the value of L_2/R_E.

The way in which the neutron flux distribution changes as the control rods are inserted will become important when the temperature distribution is analyzed in later chapters. Evaluating the neutron flux in the above solution and normalizing each distribution in the z direction by the maximum value of ϕ occurring within it (denoted by ϕ_M) the distribution becomes:

$$\begin{aligned} \phi/\phi_M &= \sin\{\xi_1 z\} \quad \text{for} \quad 0 \le z \le (1-\beta)H_E \\ &= \frac{\sin\{\xi_1(1-\beta)H_E\}}{\{e^{\xi_2\beta H_E} - e^{-\xi_2\beta H_E}\}}\left\{e^{\xi_2(H_E - z)} - e^{-\xi_2(H_E - z)}\right\} \\ &\quad \text{for} \quad (1-\beta)H_E \le z \le H_E \end{aligned} \tag{12}$$

Typical examples of these neutron flux distributions are shown in figure 3; as the fractional insertion, β, increases note how the neutron flux in the region of insertion decreases and the distribution becomes skewed toward the lower part of the core.

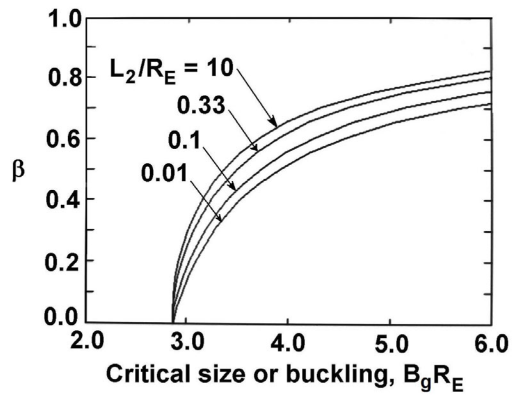

Figure 2: The critical non-dimensional size or geometric buckling, $B_g R_E$, as a function of the fractional control rod insertion, β, for a cylindrical reactor with $H_E/R_E = 2.0$ and several values of L_2/R_E as indicated.

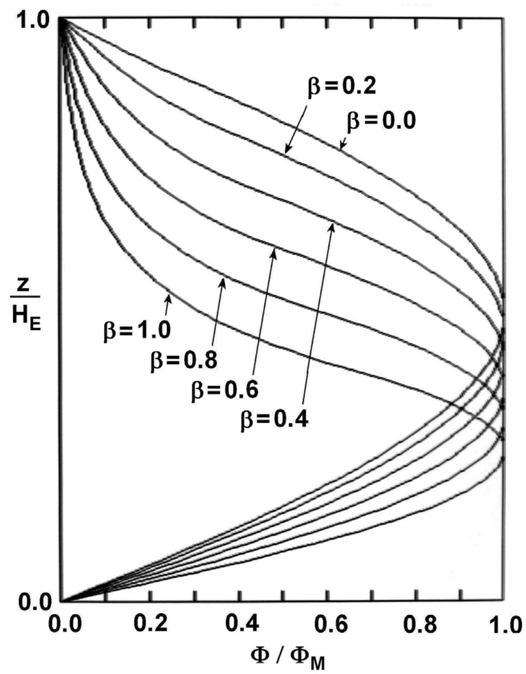

Figure 3: The change in the shape of the axial distribution of the neutron flux, ϕ (normalized by the maximum neutron flux, ϕ_M), with fractional control rod insertion, β, for the case of $H_E/R_E = 2.0$ and $L_2/R_E = 0.36$.