### 3.7.3 Effect of a reflector on a cylindrical reactor

As a second example of the effect of a reflector, consider the cylindrical reactor of section 3.7 .1 surrounded at larger radii by a reflector as shown in figure 1 (for simplicity it is assumed that vacuum conditions pertain at both ends of the core and the reflector). Then, as in section 3.7.1, the appropriate, non-singular solution to equation 5 , section 3.6.3, for the neutron flux in the core is

$$
\begin{equation*}
\phi=C \cos \left(\frac{\pi z}{H_{E}}\right) J_{0}\left(\xi_{1} r\right) \tag{1}
\end{equation*}
$$

where $H_{E}=H+1 / D$ as before and $C$ and $\xi_{1}$ are constants as yet undetermined. Turning now to the solution for equation 7 , section 3.7.1, in the cylindrical


Figure 1: Cylindrical reactor with reflector.
annulus occupied by the reflector it is assumed, for simplicity, that this extends all the way from $r=R$ to $r \rightarrow \infty$ and that the reflector has the same height $H_{E}$ as the core. Then, omitting terms that are singular as $r \rightarrow \infty$, the appropriate solution to equation 7 , section 3.7.1, in the reflector is

$$
\begin{equation*}
\phi_{R}=C_{R} \cos \left(\frac{\pi z}{H_{E}}\right) K_{0}\left(\xi_{2} r\right) \tag{2}
\end{equation*}
$$

where $\xi_{2}$ is to be determined and $K_{0}$ is the modified Bessel function. Applying the boundary conditions at the core-reflector interface, $r=R$, (equations 8, section 3.7.1) yields the relations

$$
\begin{equation*}
C J_{0}\left(\xi_{1} R\right)=C_{R} K_{0}\left(\xi_{2} R\right) \quad \text { and } \quad \xi_{1} D C J_{1}\left(\xi_{1} R\right)=\xi_{2} D_{R} C_{R} K_{1}\left(\xi_{2} R\right) \tag{3}
\end{equation*}
$$

and, upon elimination of $C_{R} / C$, these yield

$$
\begin{equation*}
D \xi_{1} J_{1}\left(\xi_{1} R\right) K_{0}\left(\xi_{2} R\right)=D_{R} \xi_{2} K_{1}\left(\xi_{2} R\right) J_{0}\left(\xi_{1} R\right) \tag{4}
\end{equation*}
$$

and, in a manner analogous to equation 7 , section 3.7 .2 , this equation must be solved numerically to determine $R$, the critical size of such a cylindrical reactor. The corresponding solutions for a reflector with a finite outer radius or with a reflector at the ends, though algebraically more complicated, are conceptually similar.

