3.7.1 Spherical and Cylindrical Reactors

Notwithstanding the limitations of the one-speed diffusion theory, it is appropriate to pursue further reactor analyses because they yield qualitatively useful results and concepts. As previously mentioned, the Helmholtz diffusion equation 5, section 3.6.3, permits solutions by separation of variables in many simple coordinate systems. Perhaps the most useful are the solutions in cylindrical coordinates since this closely approximates the geometry of most reactor cores.

However, the solutions in spherical coordinates are also instructive and it is useful to begin with these. It is readily seen that, in a spherically symmetric core (radial coordinate, r) the solution to equation 5, section 3.6.3, takes the form

$$\phi = C_1 \frac{\sin B_g r}{r} + C_2 \frac{\cos B_g r}{r} \tag{1}$$

where C_1 and C_2 are constants to be determined. For ϕ to be finite in the center, C_2 must be zero. The boundary condition at the surface, $r = R$, of this spherical reactor follows from the assumption that it is surrounded by a vacuum. Consequently the appropriate boundary condition is given by equation 5, section 3.6.1, or more conveniently $\phi = 0$ at the extrapolated boundary at $r = R_E = R + 1/2D$. Thus

$$\sin B_g R_E = 0 \qquad \text{or} \qquad B_g R_E = n\pi \tag{2}$$

where n is an integer. Since B_g and n are positive and ϕ cannot be negative anywhere within the core, the only acceptable, non-trivial value for n is unity and therefore

$$R_E = \pi/B_g \qquad \text{and thus} \qquad R = \pi/B_g - 1/2D \tag{3}$$

Therefore, $R = R_C = \pi/B_m - 1/2D$ is the critical size of a spherical reactor, that is to say the only size for which a steady neutron flux state is possible for the given value of the material buckling, B_m. It is readily seen from equation 1, section 3.5, that $\partial\phi/\partial t$ will be positive if $R > R_C$ and that the neutron flux will then grow exponentially with time. Conversely when $R < R_C$, the neutron flux will decay exponentially with time.

In summary, the neutron flux solution for the steady state operation of a spherically symmetric reactor is

$$\phi = C_1 \frac{\sin B_g r}{r} \qquad \text{for} \qquad 0 < r < R_C \tag{4}$$

Note that the neutron flux is largest in the center and declines near the boundary due to the increased leakage. Also note that though the functional form of the neutron flux variation has been determined, the magnitude of the neutron flux as defined by C_1 remains undetermined since the governing equation and boundary conditions are all homogeneous in ϕ.

Most common reactors are cylindrical and so, as a second example, it is useful to construct the solution for a cylinder of radius, R, and axial length, H, using

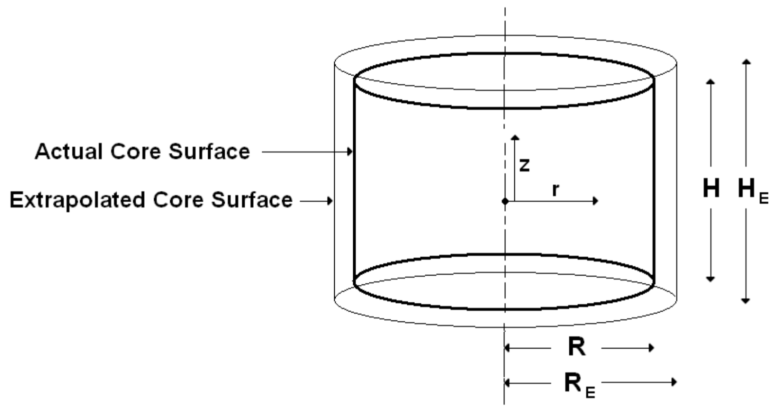

Figure 1: Sketch of a simple cylindrical reactor.

cylindrical coordinates, (r, θ, z), with the origin at the mid-length of the core. It is assumed that the reactor is homogeneous so that there are no gradients in the θ direction and that both the sides and ends see vacuum conditions. Again it is convenient to apply the condition $\phi = 0$ on extrapolated boundary surfaces at $r = R_E = R + 1/2D$ and at $z = \pm H_E/2 = \pm(H/2 + 1/2D)$ as depicted in figure 1. Obtaining solutions to equation 5, section 3.6.3, by separation of variables and eliminating possible solutions that are singular on the axis, it is readily seen that the neutron flux has the form:

$$\phi = C_1 \cos\left(\frac{\pi z}{H_E}\right) J_0\left(\frac{2.405 r}{R_E}\right) \tag{5}$$

where, as before, C_1 is an undetermined constant and $J_0()$ is the zero-order Bessel function of the first kind (2.405 is the argument that gives the first zero of this function). As in the spherical case the higher order functions are rejected since they would imply negative neutron fluxes within the cylindrical reactor. Substituting this solution into the governing equation 5, section 3.6.3, yields the expression that determines the critical size of this cylindrical reactor namely

$$\left(\frac{\pi}{H_E}\right)^2 + \left(\frac{2.405}{R_E}\right)^2 = B_g^2 \tag{6}$$

If H_E and R_E are such that the left-hand side is greater than the material buckling, B_m^2, then the reactor is supercritical and the neutron flux will grow exponentially with time; if the left hand side is less than B_m^2 the flux will decay exponentially. In the critical reactor ($B_g^2 = B_m^2$), the neutron flux is greatest in the center and decays toward the outer radii or the ends since the leakage is greatest near the boundaries.

These two examples assumed homogeneous reactors surrounded by vacuum conditions. There are a number of ways in which these simple solutions can be modified in order to incorporate common, practical variations. Often the reactor core is surrounded, not by a vacuum, but by a *blanket* of moderator that causes some of the leaking neutrons to be scattered back into the core. Such a blanket is called a reflector; examples of diffusion theory solutions that incorporate the effect of a reflector are explored in the next section. Another practical modification is to consider two core regions rather than one in order to model that region into which control rods have been inserted. Section 3.7.4 includes an example of such a two-region solution.