3.6.4 Two-speed diffusion theory

The next level of approximation is to assume that there are two speeds of neutrons, namely one group of fast neutrons that are all traveling at the same speed and a second group of thermal neutrons also all traveling with the same speed. These two neutron fluxes will be denoted by ϕ_F and ϕ_T respectively and the slowing down from the fast to the thermal neutron group will be modeled by defining a macroscopic cross-section for slowing denoted by Σ_{FT}. Focusing first on the diffusion equation for the thermal neutron flux, ϕ_T, the source term in equation 1, section 3.6.3, represents the rate of supply of thermal neutrons due to the slowing down of fast neutrons and will therefore be given by $P_F \Sigma_{FT} \phi_F$ and the first of the two coupled differential equations that constitute the two-speed diffusion model becomes

$$\bigtriangledown^2 \ \phi_T - \frac{\phi_T}{L_T^2} = - \ \frac{P_F \Sigma_{FT}}{D_T} \ \phi_F \tag{1}$$

where L_T is the neutron diffusion length for the thermal neutrons.

Turning to the fast neutrons, the absorption of fast neutrons will be neglected in comparison with the slowing down. Then Σ_{FT} is analogous to Σ_a for the thermal neutrons. Hence a neutron diffusion length for the fast neutrons can be defined as $L_F^2 = D_F / \Sigma_{FT}$. It remains to establish the source term for the fast neutrons, the rate at which fast neutrons are produced by fission. Beginning with the equation 1, section 3.6.3, for S from the one-speed model, it is reasonable to argue that the appropriate ϕ in this two-speed model is ϕ_T / P_F or the flux of thermal neutrons causing fission in the absence of resonant absorption. Thus the source term in the fast neutron continuity equation will be $k_\infty \Sigma_a \phi_T / P_F$ and the second of the two coupled differential equations, namely that for the fast neutrons, becomes

$$\bigtriangledown^2 \ \phi_F - \frac{\phi_F}{L_F^2} = - \frac{k_\infty \Sigma_a}{D_F P_F} \phi_T \tag{2}$$

Since $\Sigma_a = D_T / L_T^2$ and $\Sigma_{FT} = D_F / L_F^2$ the two equations 2 and 1 may be written as

$$\bigtriangledown^2 \ \phi_F - \frac{\phi_F}{L_F^2} = - \frac{D_T}{D_F} \frac{k_\infty}{P_F L_T^2} \phi_T \tag{3}$$

$$\bigtriangledown^2 \ \phi_T - \frac{\phi_T}{L_T^2} = - \frac{D_F}{D_T} \frac{P_F}{L_F^2} \phi_F \tag{4}$$

The solution of these coupled differential equations is simpler than might first appear for it transpires that the solutions for ϕ_F and ϕ_T take the same functional form as those of the one-speed equation 5, section 3.6.3, provided the constant B_g is appropriately chosen. This tip-off suggests a solution of the form

$$\bigtriangledown^2 \ \phi_F = -B_g^2 \phi_F \quad ; \quad \bigtriangledown^2 \ \phi_T = -B_g^2 \phi_T \tag{5}$$

Substituting into equations 3 and 4, it transpires that B_g^2 must satisfy

$$(1 + B_g^2 L_T^2)(1 + B_g^2 L_F^2) = k_\infty \tag{6}$$

Consequently solutions to the two-speed diffusion equations are of the form given in equations 5 where B_g^2 must satisfy the quadratic relation 6. It follows from equation 6 that there are two possible values for B_g^2. In the cases of interest $k_\infty > 1$ and therefore one of the values of B_g^2 is positive and the other is negative. In most circumstances (though not all) the component of the solution arising from the negative root can be neglected or eliminated leaving only the component resulting from the positive root. Moreover in the common circumstance in which k_∞ is just slightly greater than unity, the positive root is given approximately by

$$B_g^2 \approx k_\infty/(L_T^2 + L_F^2) \tag{7}$$

Thus both the fission and thermal neutrons are governed by the same diffusion equation as in the one-speed diffusion theory and with a geometric buckling that is a minor modification of that used in the earlier theory. It follows that the one-speed solutions that will be detailed in sections 3.7.1 to 3.7.4 can be readily adapted to two-speed solutions.