3.6.3 Steady State One-Speed Diffusion Theory

The most elementary application of diffusion theory is to the steady state operation of a reactor in which the neutron flux is neither increasing or decreasing in time. Then, with the time-derivative term set equal to zero, the one-speed diffusion equation 1, section 3.6.2, becomes:

$$-D \bigtriangledown^2 \phi = S - \Sigma_a \phi \tag{1}$$

assuming that the diffusion coefficient, D, is uniform throughout the reactor. Here the left-hand side is the flux of neutrons out of the control volume per unit volume. Thus, in steady state, this must be equal to the right-hand side, the excess of the rate of neutron production over the rate of neutron absorption per unit volume. This excess is a basic property of the fuel and other material properties of the reactor, in other words a *material property* as defined in section 2.10. Furthermore, by definition this excess must be proportional to $(k_\infty - 1)$ (not $(k - 1)$ since the loss to the surroundings is represented by the left hand side of equation 1). Consequently it follows that the appropriate relation for the source term is

$$S = k_\infty \Sigma_a \phi \tag{2}$$

so that, using the relation 2, section 3.6.2, the one-speed diffusion equation, equation 1, can be written as

$$\bigtriangledown^2 \phi + \frac{(k_\infty - 1)}{L^2} \phi = 0 \tag{3}$$

The material parameter $(k_\infty - 1)/L^2$ is represented by B_m^2 and, as indicated in section 2.10, is called the *material buckling*:

$$B_m^2 = \frac{(k_\infty - 1)\Sigma_a}{D} = \frac{(k_\infty - 1)}{L^2} \tag{4}$$

where $(B_m)^{-1}$ has the dimensions of length. Thus the diffusion equation 3, section 3.6.3 that applies to the steady state operation of the reactor is written as

$$\bigtriangledown^2 \phi + B_m^2 \phi = 0 \tag{5}$$

Equation 5 (or 3) is Helmholtz' equation. It has convenient solutions by separation of variables in all the simple coordinate systems. Later detailed eigensolutions to equation 5 will be examined for various reactor geometries. These solutions demonstrate that, in any particular reactor geometry, solutions that satisfy the necessary boundary conditions only exist for specific values (eigenvalues) of the parameter B_m^2. These specific values are called the *geometric buckling* and are represented by B_g^2; as described in section 2.10 the values of B_g^2 are only functions of the geometry of the reactor and not of its neutronic parameters. It follows that steady state critical solutions only exist when

$$B_m^2 = B_g^2 \tag{6}$$

and this defines the conditions for steady state criticality in the reactor. Moreover it follows that supercritical and subcritical conditions will be defined by the inequalities

$$\text{Subcritical condition:} \quad B_m^2 < B_g^2 \tag{7}$$

$$\text{Supercritical condition:} \quad B_m^2 > B_g^2 \tag{8}$$

since, in the former case, the production of neutrons is inadequate to maintain criticality and, in the latter, it is in excess of that required.

As a footnote, the multiplication factor, k, in the finite reactor can be related to the geometric buckling as follows. From equation 1, section 2.3.1, k may be evaluated as

$$k = \frac{\text{Rate of neutron production}}{\text{Sum of rates of neutron absorption and escape}} \tag{9}$$

and, in the diffusion equation solution, the rate of escape to the surroundings is represented by $-D \bigtriangledown^2 \phi$ and therefore by $DB_g^2\phi$. The corresponding rate of production is given by $Dk_\infty\phi/L^2$ and the rate of neutron absorption by $D\phi/L^2$. Substituting these expressions into the equation 9 it is observed that in steady state operation

$$k = \frac{Dk_\infty\phi/L^2}{(D\phi/L^2) + DB_g^2\phi} = \frac{k_\infty}{(1 + B_g^2 L^2)} \tag{10}$$