3.2 Neutron density and neutron flux

It is convenient to begin by defining several characteristic features of neutron transport and by introducing the concept of neutron density, N, a measure of the number of free neutrons per unit volume. Of course, this may be a function of time, t, and of position, x_i, within the core. Furthermore, these neutrons may have a range of different energies, E, and the number traveling in a particular angular direction, Ω_j (a unit vector), may have a different density than those traveling in another direction. Consequently, to fully describe the neutron density N must be considered to be a function of x_i, t, E and Ω_j and the number of neutrons in a differential volume dV that have energies between E and $E + dE$ and are traveling within the small solid angle, $d\Omega$, around the direction Ω_j would be

$$N(x_i, t, E, \Omega_j)\ dV\ dE\ d\Omega \qquad (1)$$

Consequently N has units of number per unit volume per unit energy per unit solid angle. Even for a simple core geometry N, when discretized, is a huge matrix, especially since the energy spectrum may require very fine discretization in order to accurately portray the variation with E (see, for example, figure 1).

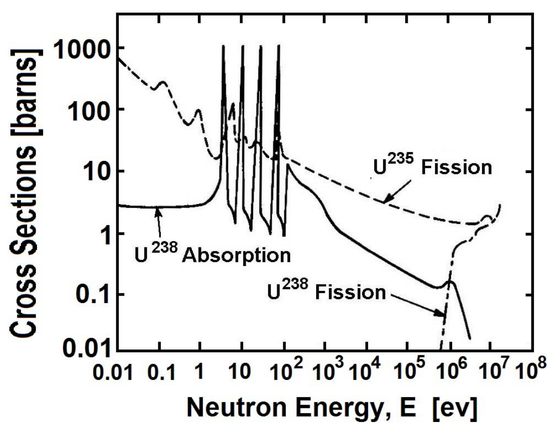

Figure 1: Qualitative representations of how the fission cross-sections for ^{235}U and ^{238}U as well as the absorption cross-section for ^{238}U vary with the neutron energy.

Denoting the magnitude of the neutron velocity by $\bar{u}(x_i, t, E)$ (a function of position, time and energy but assumed independent of direction, Ω_j) it is conventional to define the *angular neutron flux*, φ, by

$$\varphi(x_i, t, E, \Omega_j) = N(x_i, t, E, \Omega_j)\ \bar{u}(E) \qquad (2)$$

The conventional semantics here are somewhat misleading since φ is not a flux in the sense that term is commonly used in physics (indeed φ as defined above is a scalar whereas a conventional flux is a vector); it is perhaps best to regard φ as a convenient mathematical variable whose usefulness will become apparent later.

A more physically recognizable characteristic is the conventional vector quantity known as the *angular current density*, J_j^*, given by

$$J_j^*(x_i, t, E, \Omega_j) = \bar{u}\ \Omega_j\ N(x_i, t, E, \Omega_j) = \Omega_j\ \varphi(x_i, t, E, \Omega_j) \tag{3}$$

since $\bar{u}\Omega_j$ is the vector velocity of a neutron traveling in the direction Ω_j. This angular current density, J_j^*, might be more properly called the neutron flux but confusion would result from altering the standard semantics. The physical interpretation is that $J_j^* dE d\Omega$ is the number of neutrons (with energies between E and $E + dE$) traveling within the solid angle $d\Omega$ about the direction Ω_j per unit area normal to that direction per unit time. Note that since Ω_j is a unit vector, the magnitude of J_j^* is φ.

The above definitions allow for the fundamental quantities φ and J_j^* to vary with the angular orientation Ω_j. However it will often be assumed that these variations with orientation are small or negligible. Then integration over all orientations allows the definition of an angle-integrated neutron flux, $\phi(x_i, t, E)$ (later abbreviated to *neutron flux*), and an angle-integrated current density, $J_j(x_i, t, E)$:

$$\phi(x_i, t, E) = \int_{4\pi} \varphi(x_i, t, E, \Omega_j) d\Omega \tag{4}$$

$$J_j(x_i, t, E) = \int_{4\pi} J_k^*(x_i, t, E, \Omega_j) d\Omega \tag{5}$$

Note that if $\varphi(x_i, t, E, \Omega_j)$ and/or $J_j^*(x_i, t, E, \Omega_j)$ are isotropic and therefore independent of Ω_j then

$$\phi = 4\pi\varphi \qquad ; \qquad J_j = 4\pi J_j^* \tag{6}$$

In the simpler neutronics calculations later in this book, the *neutron flux*, ϕ, is the dependent variable normally used in the calculations.