2.3.3 Cross-sections and mean free paths

In the context of nuclear interactions or events, a cross-section is a measure of the probability of occurrence of that interaction or event. Consider, for example, a highly simplified situation in which n neutrons per cm^3, all of the same velocity, $\bar{u}$, (or energy, E) are zooming around in a reactor volume of density ρ consisting of only one type of atom of atomic weight, A. The number of atoms per gram is therefore $6.025 \times 10^{23}/A$ where 6.025×10^{23} is Avagadro's number. Hence, the number of atoms per cm^3, $\mathcal{N}$, is given by

$$\mathcal{N} = 6.025 \times 10^{23} \rho / A \tag{1}$$

In such a reactor, the rate at which the moving neutrons are colliding with atoms (assumed stationary) within each cm^3 of volume is clearly going to be proportional to $\mathcal{N}$, to n and to the velocity, $\bar{u}$, of the neutrons. The factor of proportionality, σ, or

$$\sigma = \frac{\text{Number of collisions per unit time per unit volume}}{\mathcal{N} n \bar{u}} \tag{2}$$

has units of area and is known as a cross-section. It can be visualized as the effective frontal area of the atom that would lead to the given collision rate per unit volume (zero area would, of course, not lead to any collisions). Cross-sections are measured in units called *barns* where 1 $barn = 10^{-24}\ cm^2$. They are a measure of the probability of a particular event occurring in unit volume per unit time divided by the number of collisions per unit time per unit volume as indicated in equation 2. Thus, for example, the probability of a collision causing fission is proportional to the *fission cross-section*, σ_f, the probability of a collision resulting in neutron capture or absorption is proportional to the *absorption cross-section*, σ_a, and the probability of a collision resulting in scattering is proportional to the *scattering cross-section*, σ_s.

The typical distance traveled by a neutron between such interaction events is called the mean free path, ℓ, and this is related to the cross-section as follows. Consider a given interval of time. Then the mean free path, ℓ, will be given by the total distance traveled by all neutrons in a unit volume during that time divided by the number of neutrons undergoing a particular interaction during that time. Or

$$\ell = \frac{n\bar{u}}{\mathcal{N} n \bar{u} \sigma} = \frac{1}{\mathcal{N}\sigma} \tag{3}$$

More specifically, the *fission mean free path*, $\ell_f = 1/\mathcal{N}\sigma_f$, will be the typical distance traveled by a neutron between fission events, the *absorption mean free path*, $\ell_a = 1/\mathcal{N}\sigma_a$, will be the typical distance traveled by a neutron between absorption events and so on. For this and other reasons, it is convenient to define *macroscopic cross-sections* , Σ, where $\Sigma = \mathcal{N}\sigma$; these macroscopic cross-sections therefore have units of inverse length.

Note that most of the cross-sections, σ, that are needed for reactor analysis are strong and often complicated functions of the neutron energy, E. This

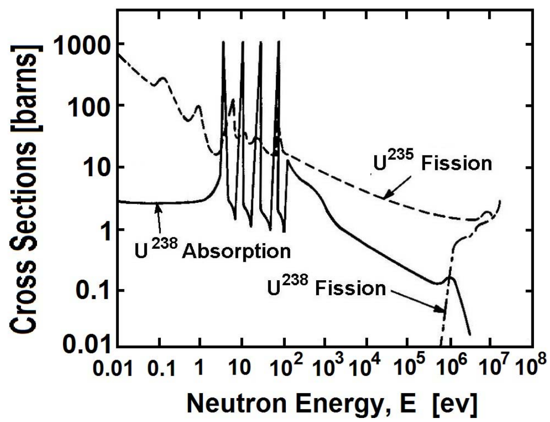

Figure 1: Qualitative representations of how the fission cross-sections for ^{235}U and ^{238}U as well as the absorption cross-section for ^{238}U vary with the neutron energy.

complicates the quantitative analyses of most reactors even when the conceptual processes are quite straightforward. Qualitative examples of how some cross-sections vary with E are included in figure 1. Rough models of how σ_a, σ_f, and σ_s depend on E (or $\bar{u}$) are as follows.

In many materials, thermal neutrons have σ_a and σ_f cross-sections that are inversely proportional the velocity, $\bar{u}$, and therefore vary like $E^{-\frac{1}{2}}$. In such materials it is conventional to use a factor of proportionality, $\sigma E^{\frac{1}{2}}$, at a reference state corresponding to a velocity of 2200 m/s (or an energy of $E = 0.0253\ eV$). The average cross-section so defined at $E = 0.0253\ eV$ is called the *thermal cross-section reduced to* $0.0253\ eV$ and will be denoted here by $\hat{\sigma}$.

Of course, some materials like ^{238}U have strong absorption peaks or resonances near particular energies and, in these, the model described above requires modification. This is often effected by supplementing the $E^{-\frac{1}{2}}$ dependence with one or more resonance peaks. Another useful observation is that scattering cross-sections, σ_s, are often independent of E except at high velocities and can therefore be modeled using a single uniform value.