

## System Structure

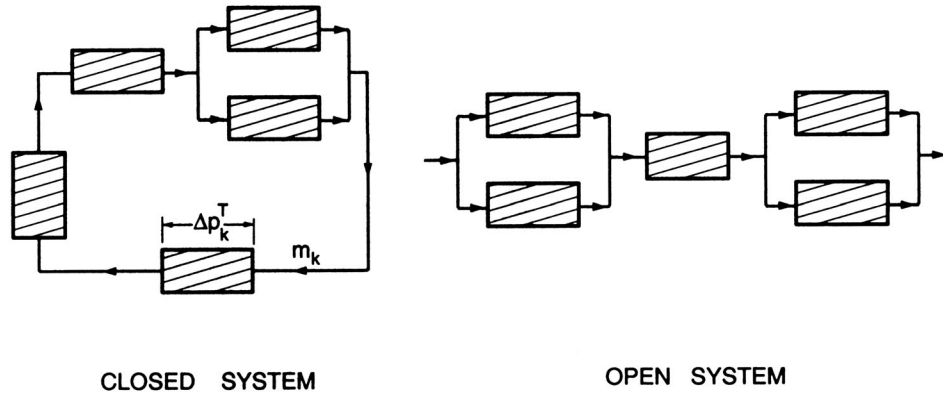


Figure 1: Flow systems broken into components.

In the discussion and analysis of system stability, we shall consider that the system has been divided into its components, each identified by its index,  $k$ , as shown in figure 1 where each component is represented by a box. The connecting lines do not depict lengths of pipe which are themselves components. Rather the lines simply show how the components are connected. More specifically they represent specific locations at which the system has been divided up; these points are called the nodes of the system and are denoted by the index,  $i$ .

Typical and common components are pipeline sections, valves, pumps, turbines, accumulators, surge tanks, boilers, and condensers. They can be connected in series and/or in parallel. Systems can be either open loop or closed loop as shown in figure 1. The mass flow rate through a component will be denoted by  $\dot{m}_k$  and the change in the total head of the flow across the component will be denoted by  $\Delta p_k^T$  defined as the total pressure at inlet minus that at discharge. (When the pressure ratios are large enough so that the

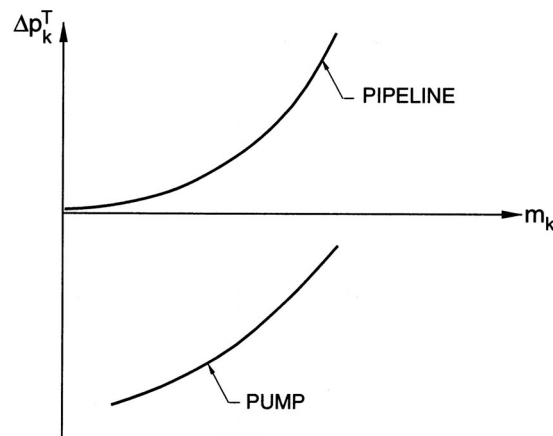


Figure 2: Typical component characteristics,  $\Delta p_k^T(\dot{m}_k)$ .

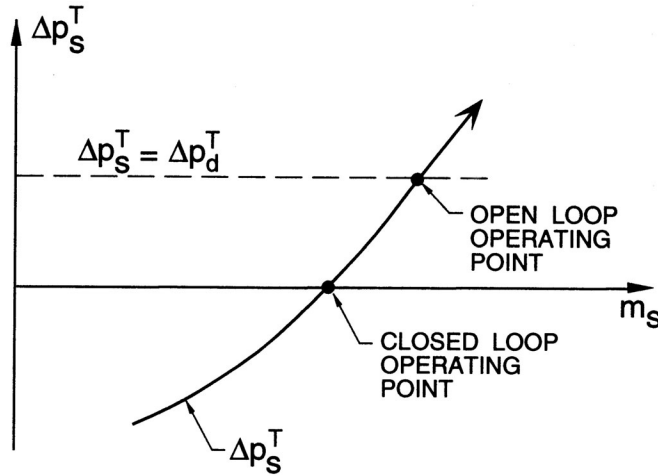


Figure 3: Typical system characteristic,  $\Delta p_s^T(m_s)$ , and operating point.

compressibility of one or both of the phases must be accounted for, the analysis can readily be generalized by using total enthalpy rather than total pressure.) Then, each of the components considered in isolation will have a performance characteristic in the form of the function  $\Delta p_k^T(\dot{m}_k)$  as depicted graphically in figure 2. We shall see that the shapes of these characteristics are important in identifying and analysing system instabilities. Some of the shapes are readily anticipated. For example, a typical single phase flow pipe section (at higher Reynolds numbers) will have a characteristic that is approximately quadratic with  $\Delta p_k^T \propto \dot{m}_k^2$ . Other components such as pumps, compressors or fans may have quite non-monotonic characteristics. The slope of the characteristic,  $R_k^*$ , where

$$R_k^* = \frac{1}{\rho g} \frac{d\Delta p_k^T}{d\dot{m}_k} \quad (\text{Nrb1})$$

is known as the component resistance. However, unlike many electrical components, the resistance of most hydraulic components is almost never constant but varies with the flow,  $\dot{m}_k$ .

Components can readily be combined to obtain the characteristic of groups of neighboring components or the complete system. A parallel combination of two components simply requires one to add the flow rates at the same  $\Delta p^T$ , while a series combination simply requires that one add the  $\Delta p^T$  values of the two components at the same flow rate. In this way one can synthesize the total pressure drop,  $\Delta p_s^T(\dot{m}_s)$ , for the whole system as a function of the flow rate,  $\dot{m}_s$ . Such a system characteristic is depicted in figure 3. For a closed system, the equilibrium operating point is then given by the intersection of the characteristic with the horizontal axis since one must have  $\Delta p_s^T = 0$ . An open system driven by a total pressure difference of  $\Delta p_d^T$  (inlet total pressure minus discharge) would have an operating point where the characteristic intersects the horizontal line at  $\Delta p_s^T = \Delta p_d^T$ . Since these are trivially different we can confine the discussion to the closed loop case without any loss of generality.

In many discussions, this system equilibrium is depicted in a slightly different but completely equivalent way by dividing the system into two series elements, one of which is the *pumping* component,  $k = \text{pump}$ , and the other is the *pipeline* component,  $k = \text{line}$ . Then the operating point is given by the intersection of the *pipeline* characteristic,  $\Delta p_{\text{line}}^T$ , and the *pump* characteristic,  $-\Delta p_{\text{pump}}^T$ , as shown graphically in figure 4. Note that since the total pressure increases across a pump, the values of  $-\Delta p_{\text{pump}}^T$  are normally positive. In most single phase systems, this depiction has the advantage that one can usually construct a series of quadratic *pipeline* characteristics depending on the valve settings. These *pipeline* characteristics are usually simple quadratics. On the other hand the pump or compressor characteristic can be quite complex.

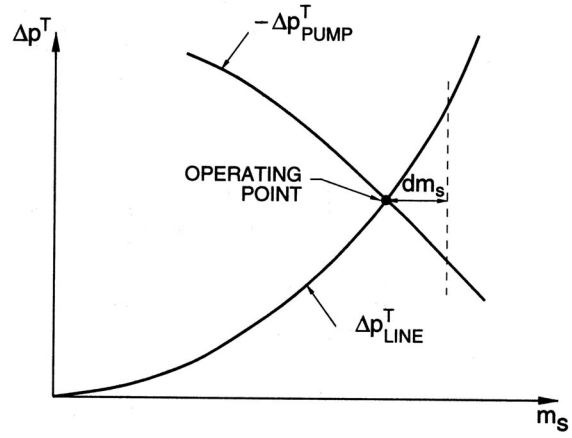


Figure 4: Alternate presentation of figure 3.