Averaging Contributions to the Mean Motion

Thus far we have discussed only those additional terms introduced as a result of the fact that the gradient of the average may differ from the average of the gradient. Inspection of the form of the basic equations (for example the continuity equation (Nbb2) or the momentum equation (Nbe5)) readily demonstrates that additional averaging terms will be introduced because the average of a product is different from the product of averages. In single phase flows, the *Reynolds stress terms* in the averaged equations of motion for turbulent flows are a prime example of this phenomenon. We will use the name *quadratic rectification terms* to refer to the appearance in the averaged equations of motion of the mean of two fluctuating components of velocity and/or volume fraction. Multiphase flows will, of course, also exhibit conventional Reynolds stress terms when they become turbulent (see section (Nc) for more on the complicated subject of turbulence in multiphase flows). But even multiphase flows that are not turbulent in the strictest sense will exhibit variations in the velocities due the flows around particles and these variations will yield quadratic rectification terms. These must be recognized and modeled when considering the effects of locally non-uniform and unsteady velocities on the equations of motion. Much more has to be learned of both the laminar and turbulent quadratic rectification terms before these can be confidently incorporated in model equations for multiphase flow. Both experiments and computer simulation will be valuable in this regard.

One simpler example in which the fluctuations in velocity have been measured and considered is the case of concentrated granular flows in which direct particle-particle interactions create particle velocity fluctuations. These particle velocity fluctuations and the energy associated with them (the so-called granular temperature) have been studied both experimentally and computationally (see section (Np)) and their role in the effective continuum equations of motion is better understood than in more complex multiphase flows.

With two interacting phases or components, the additional terms that emerge from an averaging process can become extremely complex. In recent decades a number of valiant efforts have been made to codify these issues and establish at least the forms of the important terms that result from these interactions. For example, Wallis (1991) has devoted considerable effort to identify the inertial coupling of spheres in inviscid, locally irrotational flow. Arnold, Drew and Lahey (1989) and Drew (1991) have focused on the application of cell methods (see section (Nek)) to interacting multiphase flows. Both these authors as well as Sangani and Didwania (1993) and Zhang and Prosperetti (1994) have attempted to include the fluctuating motions of the particles (as in granular flows) in the construction of equations of motion for the multiphase flow; Zhang and Prosperetti also provide a useful comparative summary of these various averaging efforts. However, it is also clear that these studies have some distance to go before they can be incorporated into any real multiphase flow prediction methodology.