Kinematic Shock Waves

The results of section (Nsb) will now be extended by considering the relations for a finite kinematic wave or shock. As sketched in figure 1 the conditions ahead of the shock will be denoted by the subscript 1 and the conditions behind the shock by the subscript 2. Two questions must be asked. First, does such a structure exist and, if so, what is its propagation velocity, u_s? Second, is the structure stable in the sense that it will persist unchanged for a significant time? The first question is addressed in this section, the second question in the section that follows. For the sake of simplicity, any differences in the component densities across the shock will be neglected; it is also assumed that no exchange of mass between the phases or components occurs within the shock. In section (Nsg), the role that might be played by each of these effects will be considered.

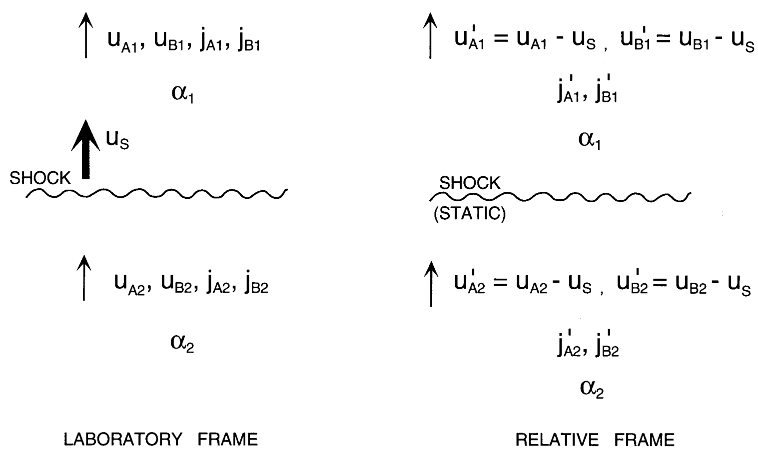

Figure 1: Velocities and volume fluxes associated with a kinematic shock in the laboratory frame (left) and in a frame relative to the shock (right).

To determine the speed of the shock, u_s, it is convenient to first apply a Galilean transformation to the situation on the left in figure 1 so that the shock position is fixed (the diagram on the right in figure 1). In this relative frame we denote the velocities and fluxes by the prime. By definition it follows that the fluxes relative to the shock are related to the fluxes in the original frame by

$$j'_{A1} = j_{A1} - \alpha_1 u_s \quad ; \quad j'_{B1} = j_{B1} - (1-\alpha_1)u_s \tag{Nse1}$$

$$j'_{A2} = j_{A2} - \alpha_2 u_s \quad ; \quad j'_{B2} = j_{B2} - (1-\alpha_2)u_s \tag{Nse2}$$

Then, since the densities are assumed to be the same across the shock and no exchange of mass occurs, conservation of mass requires that

$$j'_{A1} = j'_{A2} \quad ; \quad j'_{B1} = j'_{B2} \tag{Nse3}$$

Substituting the expressions (Nse1) and (Nse2) into equations (Nse3) and replacing the fluxes, j_{A1}, j_{A2}, j_{B1} and j_{B2}, using the identities (Nsb4) involving the total flux, j, and the drift fluxes, j_{AB1} and j_{AB2}, we obtain the following expression for the shock propagation velocity, u_s:

$$u_s = j + \frac{j_{AB2} - j_{AB1}}{\alpha_2 - \alpha_1} \tag{Nse4}$$

where the total flux, j, is necessarily the same on both sides of the shock. Now, if the drift flux is a function only of α it follows that this expression can be written as

$$u_s = j + \frac{j_{AB}(\alpha_2) - j_{AB}(\alpha_1)}{\alpha_2 - \alpha_1} \tag{Nse5}$$

Note that, in the limit of a small amplitude wave ($\alpha_2 \rightarrow \alpha_1$) this reduces, as it must, to the expression (Nsb7) for the speed of an infinitesimal wave.

So now we add another aspect to figure **??** and indicate that, as a consequence of equation (Nse5), the speed of a shock between volume fractions α_2 and α_1 is given by the slope of the line connecting those two points on the drift flux curve (plus j).