Size Distributions

In many multiphase flow contexts we shall make the simplifying assumption that all the disperse phase particles (bubbles, droplets or solid particles) have the same size. However in many natural and technological processes it is necessary to consider the distribution of particle size. One fundamental measure of this is the size distribution function, $N(v)$, defined such that the number of particles in a unit volume of the multiphase mixture with volume between v and $v+dv$ is $N(v)dv$. For convenience, it is often assumed that the particles size can be represented by a single linear dimension (for example, the diameter, D, or radius, R, in the case of spherical particles) so that alternative size distribution functions, $N'(D)$ or $N''(R)$, may be used. Examples of size distribution functions based on radius are shown in figures 1 and 2.

Often such information is presented in the form of cumulative number distributions. For example the cumulative distribution, $N^*(v^*)$, defined as

$$N^*(v^*) = \int_0^{v^*} N(v)dv \qquad \text{(Nad1)}$$

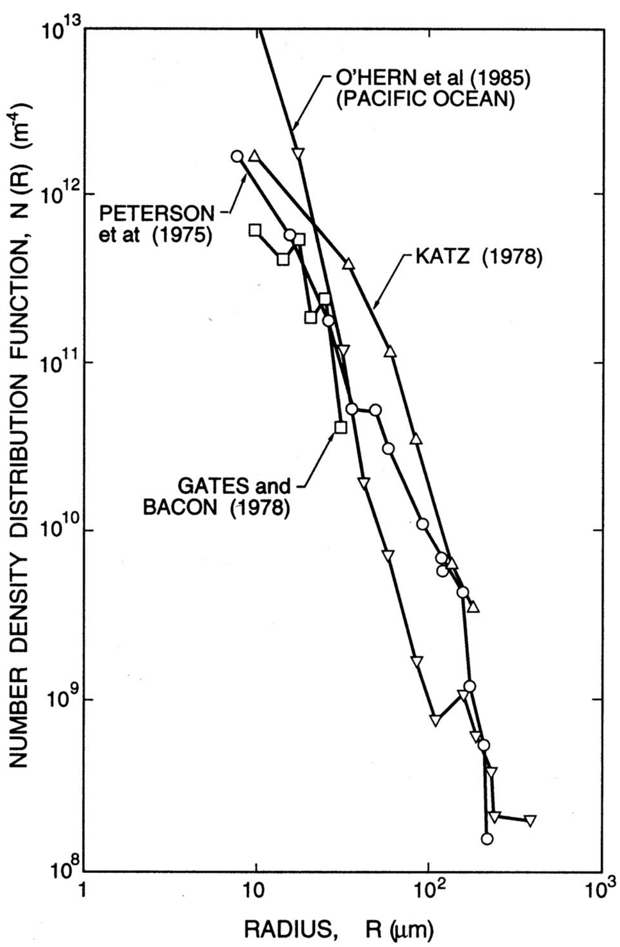

Figure 1: Measured size distribution functions for small bubbles in three different water tunnels (Peterson *et al.* 1975, Gates and Bacon 1978, Katz 1978) and in the ocean off Los Angeles, Calif. (O'Hern *et al.* 1985).

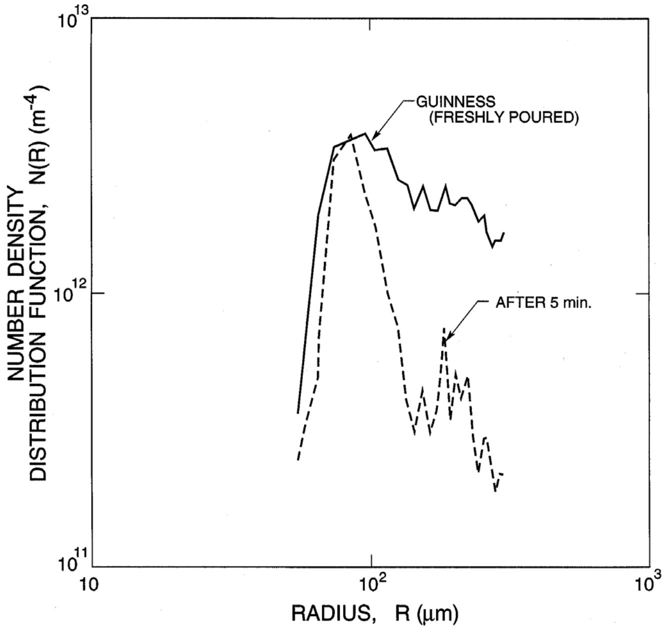

Figure 2: Size distribution functions for bubbles in freshly poured Guinness and after five minutes. Adapted from Kawaguchi and Maeda (2003).

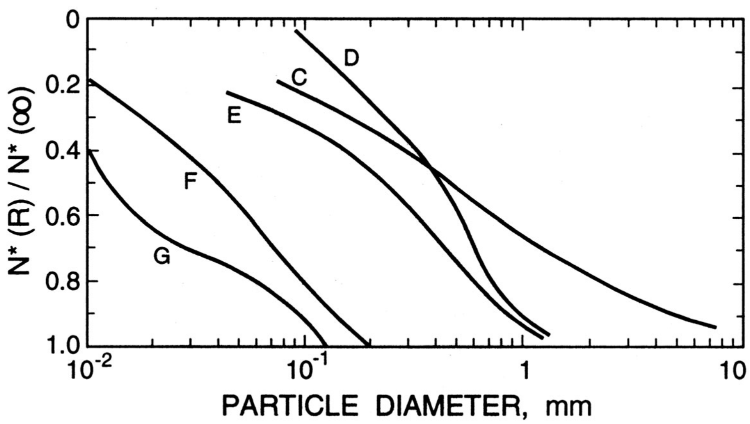

Figure 3: Cumulative size distributions for various coal slurries. Adapted from Shook and Roco (1991).

is the total number of particles of volume less than v^*. Examples of cumulative distributions (in this case for coal slurries) are shown in figure 3.

In these disperse flows, the evaluation of global quantities or characteristics of the disperse phase will clearly require integration over the full range of particle sizes using the size distribution function. For example, the volume fraction of the disperse phase, α_D, is given by

$$\alpha_D = \int_0^\infty v \; N(v) dv = \frac{\pi}{6} \int_0^\infty D^3 \; N'(D) dD \tag{Nad2}$$

where the last expression clearly applies to spherical particles. Other properties of the disperse phase or of the interactions between the disperse and continuous phases can involve other moments of the size distribution function (see, for example, Friedlander 1977). This leads to a series of mean diameters (or sizes in the case of non-spherical particles) of the form, D_{jk}, where

$$D_{jk} = \left[\frac{\int_0^\infty D^j \; N'(D) dD}{\int_0^\infty D^k \; N'(D) dD} \right]^{\frac{1}{j-k}} \tag{Nad3}$$

A commonly used example is the *mass mean* diameter, D_{30}. On the other hand processes that are controlled by particle surface area would be characterized by the *surface area mean* diameter, D_{20}. The surface area mean diameter would be important, for example, in determining the exchange of heat between the phases or the rates of chemical interaction at the disperse phase surface. Another measure of the average size that proves useful in characterizing many disperse particulates is the Sauter mean diameter, D_{32}. This is a measure of the ratio of the particle volume to the particle surface area and, as such, is often used in characterizing particulates (see, for example, section (Np)).