# Flow Regimes

As pointed out by Campbell (2002), given a particle interaction model (such as that described in section (Npb)) characterized by a set of parameters like $(K, \epsilon, \mu^*)$, it follows from dimensional analysis that the stress, $\tau$, in a typical shearing flow with a shear rate, $\dot{\gamma}$, and a solids volume fraction, $\alpha$, will be a function of the particle interaction parameters plus $(D, \rho_S, \alpha, \dot{\gamma})$ where the particle density $\rho_S$ has been used instead of the particle mass, $m_p$. Applying dimensional analysis to this function it follows that the dimensionless stress, $\tau D/K$, must be a function of the following dimensionless quantities:

$$\frac{\tau D}{K} = f\left(\alpha, \mu^*, \epsilon, \frac{K}{\rho_S D^3 \dot{\gamma}^2}\right) \tag{Npd1}$$

Alternatively one could also use a different form for the non-dimensional stress, namely $\tau/\rho_S D^2 \dot{\gamma}^2$, and express this as a function of the same set of dimensionless quantities.

Such a construct demonstrates the importance in granular flows of the parameter, $K/\rho_S D^3 \dot{\gamma}^2$, which is the square of the ratio of the typical time associated with the shearing, $t_{shear} = 1/\dot{\gamma}$, to a typical collision time, $(m_p/K)^{\frac{1}{2}}$. The shearing time, $t_{shear}$, will determine the time between collisions for a particular particle though this time will also be heavily influenced by the solids fraction, $\alpha$. The typical collision time, $(m_p/K)^{\frac{1}{2}}$, will be close to the binary collision time. From these considerations, we can discern two possible flow regimes or asymptotic flow states. The first is identified by instantaneous (and therefore, necessarily binary) collisions in which the collision time is very short compared with the shearing time so that $K/\rho_S D^3 \dot{\gamma}^2 \gg 1$. We will we refer to this as the *inertial regime*. It includes an asymptotic case called *rapid granular flows* in which the collisions are essentially instantaneous and binary. The above dimensional analysis shows that appropriate dimensionless stresses in the inertial regime take the form $\tau/\rho_S D^2 \dot{\gamma}^2$ and should be functions only of

$$\frac{\tau}{\rho_S D^2 \dot{\gamma}^2} = f(\alpha, \mu^*, \epsilon) \tag{Npd2}$$

This is the form that Bagnold (1954) surmised in his classic and much quoted paper on granular shear flows.

The second asymptotic flow regime is characterized by contact times that are long compared with the shearing time so that $K/\rho_S D^3 \dot{\gamma}^2 \ll 1$. From computer simulations Campbell (2002) finds that as $K/\rho_S D^3 \dot{\gamma}^2$ is decreased and the flow begins to depart from the inertial regime, the particles are forced to interact with a frequency whose typical time becomes comparable to the binary collision time. Consequently multiple particle interactions begin to occur and force chains begin to form. Then the dimensional analysis shows that the appropriate dimensionless stresses are $\tau D/K$ and, in this limit, these should only be functions of

$$\frac{\tau D}{K} = f(\alpha, \mu^*, \epsilon) \tag{Npd3}$$

Note that this second regime is essentially quasistatic in that the stresses do not depend on any rate quantities. Campbell refers to this as the elastic-quasistatic regime.