Linear Stability of Laminar Flow

In the preceding section we examined the behavior of small perturbations about a constant and uniform state of the mixture. The perturbation was a plane acoustic wave but the reader will recognize that an essentially similar methodology can be used (and has been) to study other types of flow involving small linear perturbations. An example is steady flow in which the deviation from a uniform stream is small. The equations governing the small deviations in a steady planar flow in, say, the (x, y) plane are then quite analogous to the equations in (x, t) derived in the preceding section.

An important example of this type of solution is the effect that dust might have on the stability of a laminar flow (for instance a boundary layer flow) and, therefore, on the transition to turbulence. Saffman (1962) explored the effect of a small volume fraction of dust on the stability of a parallel flow. As expected and as described in section (Ncb), when the response times of the particles are short compared with the typical times associated with the fluid motion, the particles simply alter the effective properties of the fluid, its effective density, viscosity and compressibility. It follows that under these circumstances the stability is governed by the effective Reynolds number and effective Mach number. Saffman considered dusty gases at low volume concentrations, α, and low Mach numbers; under those conditions the net effect of the dust is to change the density by $(1 + \alpha\rho_S/\rho_G)$ and the viscosity by $(1 + 2.5\alpha)$. The effective Reynolds number therefore varies like $(1 + \alpha\rho_S/\rho_G)/(1 + 2.5\alpha)$. Since $\rho_S \gg \rho_G$ the effective Reynolds number is increased and therefore, in the small relaxation time range, the dust is destabilizing. Conversely for large relaxation times, the dust stabilizes the flow.