Homogeneous Flow with Gas Dynamics

Though the focus in these sections is on the effect of relative motion, we must begin by examining the simplest case in which both the relative motion between the phases or components and the temperature differences between the phases or components are sufficiently small that they can be neglected. This will establish the base state that, through perturbation methods, can be used to examine flows in which the relative motion and temperature differences are small. As we established in sections (Nl), a flow with no relative motion or temperature differences is referred to as *homogeneous*. The effect of mass exchange will also be neglected in the present discussion and, in such a homogeneous flow, the governing equations, (Nnb6), (Nnb7), and (Nnb8) clearly reduce to

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{Nnc1}$$

$$\rho\left[\frac{\partial u_k}{\partial t} + u_i \frac{\partial u_k}{\partial x_i}\right] = \rho g_k - \frac{\partial p}{\partial x_k} + \frac{\partial \sigma^D_{Cki}}{\partial x_i} \tag{Nnc2}$$

$$\left[\sum_N \rho_N \alpha_N c_{vN}\right]\left\{\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i}\right\} = \sigma_{Cij}\frac{\partial u_i}{\partial x_j} \tag{Nnc3}$$

where u_i and T are the velocity and temperature common to all phases.

An important result that follows from the individual continuity equations (Nnb1) in the absence of exchange of mass ($\mathcal{I}_N = 0$) is that

$$\frac{D}{Dt}\left\{\frac{\rho_D \alpha_D}{\rho_C \alpha_C}\right\} = \frac{D\xi}{Dt} = 0 \tag{Nnc4}$$

Consequently, if the flow develops from a uniform stream in which the loading ξ is constant and uniform, then ξ is uniform and constant everywhere and becomes a simple constant for the flow. We shall confine the remarks in this section to such flows.

At this point, one particular approximation is very advantageous. Since in many applications the volume occupied by the particles is very small, it is reasonable to set $\alpha_C \approx 1$ in equation (Nnb2) and elsewhere. This approximation has the important consequence that equations (Nnc1), (Nnc2) and (Nnc3) are now those of a single phase flow of an *effective* gas whose thermodynamic and transport properties are as follows. The approximation allows the equation of state of the *effective* gas to be written as

$$p = \rho \mathcal{R} T \tag{Nnc5}$$

where $\mathcal{R}$ is the gas constant of the effective gas. Setting $\alpha_C \approx 1$, the thermodynamic properties of the effective gas are given by

$$\rho = \rho_C(1+\xi) \quad ; \quad \mathcal{R} = \mathcal{R}_C/(1+\xi)$$

$$c_v = \frac{c_{vC} + \xi c_{sD}}{1+\xi} \quad ; \quad c_p = \frac{c_{pC} + \xi c_{sD}}{1+\xi} \quad ; \quad \gamma = \frac{c_{pC} + \xi c_{sD}}{c_{vC} + \xi c_{sD}} \tag{Nnc6}$$

and the effective kinematic viscosity is

$$\nu = \mu_C/\rho_C(1+\xi) = \nu_C/(1+\xi) \tag{Nnc7}$$

Moreover, it follows from equations (Nnc6), that the relation between the isentropic speed of sound, c, in the *effective* gas and that in the continuous phase, c_C, is

$$c = c_C \left[\frac{1 + \xi c_{sD}/c_{pC}}{(1 + \xi c_{sD}/c_{vC})(1 + \xi)} \right]^{\frac{1}{2}} \tag{Nnc8}$$

It also follows that the Reynolds, Mach and Prandtl numbers for the effective gas flow, Re, M and Pr (based on a typical dimension, ℓ, typical velocity, U, and typical temperature, T_0, of the flow) are related to the Reynolds, Mach and Prandtl numbers for the flow of the continuous phase, Re_C, M_C and Pr_C, by

$$Re = \frac{U\ell}{\nu} = Re_C(1 + \xi) \tag{Nnc9}$$

$$M = \frac{U}{c} = M_C \left[\frac{(1 + \xi c_{sD}/c_{vC})(1 + \xi)}{(1 + \xi c_{sD}/c_{pC})} \right]^{\frac{1}{2}} \tag{Nnc10}$$

$$Pr = \frac{c_p \mu}{k} = Pr_C \left[\frac{(1 + \xi c_{sD}/c_{pC})}{(1 + \xi)} \right] \tag{Nnc11}$$

Thus the first step in most investigations of this type of flow is to solve for the effective gas flow using the appropriate tools from single phase gas dynamics. Here, it is assumed that the reader is familiar with these basic methods. Thus we focus on the phenomena that constitute departures from single phase flow mechanics and, in particular, on the process and consequences of relative motion or *slip*.