Flow over a Wavy Wall

A second example of this type of solution that was investigated by Zung (1967) is steady particle-laden

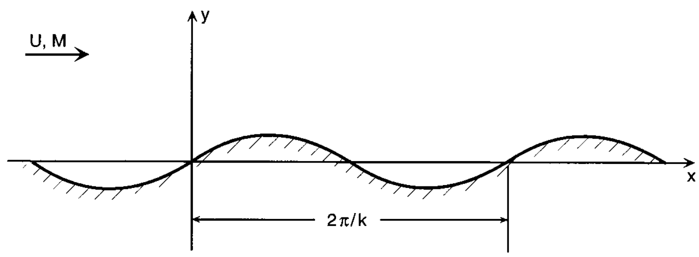

Figure 1: Schematic for flow over a wavy wall.

flow over a wavy wall of small amplitude (figure 1) so that only the terms that are linear in the amplitude need be retained. The solution takes the form

$$\exp(i\kappa_1 x - i\kappa_2 y) \tag{Nnh1}$$

where $2\pi/\kappa_1$ is the wavelength of the wall whose mean direction corresponds with the x axis and κ_2 is a complex number whose real part determines the inclination of the characteristics or Mach waves and whose imaginary part determines the attenuation with distance from the wall. The value of κ_2 is obtained in the solution from a dispersion relation that has many similarities to equation (Nnf8). Typical computations of κ_2 are presented in figure 2. The asymptotic values for large t_u that occur on the right in this figure correspond to cases in which the particle motion is constant and unaffected by the waves. Consequently, in subsonic flows ($M = U/c_C < 1$) in which there are no characteristics, the value of $Re\{\kappa_2/\kappa_1\}$ asymptotes to zero and the waves decay with distance from the wall such that $Im\{\kappa_2/\kappa_1\}$ tends to $(1-M^2)^{\frac{1}{2}}$. On the other hand in supersonic flows ($M = U/c_C > 1$) $Re\{\kappa_2/\kappa_1\}$ asymptotes to the tangent of the Mach wave angle in the gas alone, namely $(M^2-1)^{\frac{1}{2}}$, and the decay along these characteristics is zero.

At the other extreme, the asymptotic values as t_u approaches zero correspond to the case of the effective gas whose properties are given in section (Nnc). Then the appropriate Mach number, M_0, is that based on the speed of sound in the effective gas (equation (Nnc8)). In the case of figure 2, $M_0^2 = 2.4M^2$. Consequently, in subsonic flows ($M_0 < 1$), the real and imaginary parts of κ_2/κ_1 tend to zero and $(1-M_0^2)^{\frac{1}{2}}$ respectively as t_u tends to zero. In supersonic flows ($M_0 > 1$), they tend to $(M_0^2-1)^{\frac{1}{2}}$ and zero respectively.

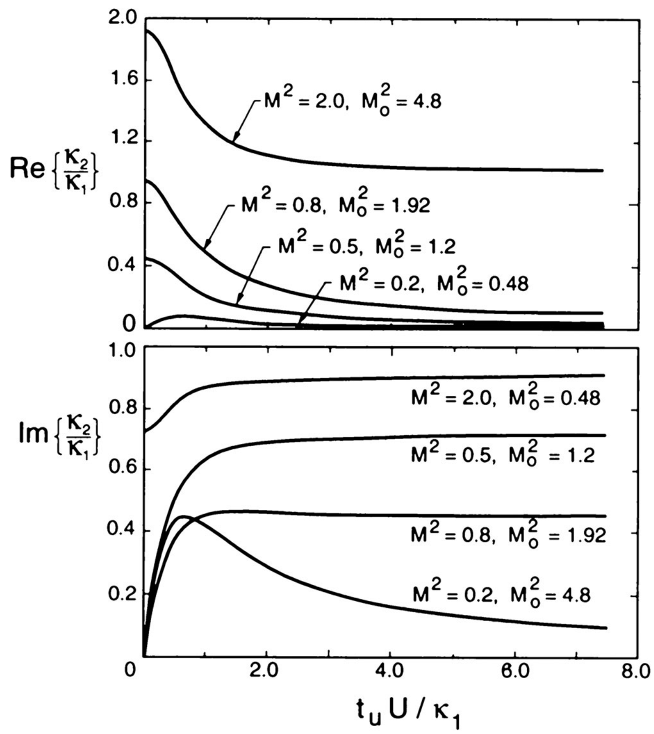

Figure 2: Typical results from the wavy wall solution of Zung (1969). Real and imaginary parts of κ_2/κ_1 are plotted against $t_u U/\kappa_1$ for various mean Mach numbers, $M = U/c_C$, for the case of $t_T/t_u = 1$, $c_{pC}/c_{sD} = 1$, $\gamma = 1.4$ and a particle loading, $\xi = 1$.