Introduction to Flows with Bubble Dynamics

In the last chapter, the analyses were predicated on the existence of an effective barotropic relation for the homogeneous mixture. Indeed, the construction of the sonic speed in sections (Nlb), (Nlc) and (Nld) assumes that all the phases are in dynamic equilibrium at all times. For example, in the case of bubbles in liquids, it is assumed that the response of the bubbles to the change in pressure, δp, is an essentially instantaneous change in their volume. In practice this would only be the case if the typical frequencies experienced by the bubbles in the flow are very much smaller than the natural frequencies of the bubbles themselves (see section (Ngj)). Under these circumstances the bubbles would behave quasistatically and the mixture would be barotropic. However, there are a number of important contexts in which the bubbles are not in equilibrium and in which the non-equilibrium effects have important consequences. One example is the response of a bubbly multiphase mixture to high frequency excitation. Another is a bubbly cavitating flow where the non-equilibrium bubble dynamics lead to shock waves with substantial noise and damage potential.

In this chapter we therefore examine some flows in which the dynamics of the individual bubbles play an important role. These effects are included by incorporating the Rayleigh-Plesset equation (Rayleigh 1917, Knapp *et al.* 1970, Brennen 1995) into the global conservation equations for the multiphase flow. Consequently the mixture no longer behaves barotropically.

Viewing these flows from a different perspective, we note that analyses of cavitating flows often consist of using a single-phase liquid pressure distribution as input to the Rayleigh-Plesset equation. The result is the history of the size of individual cavitating bubbles as they progress along a streamline in the otherwise purely liquid flow. Such an approach entirely neglects the interactive effects that the cavitating bubbles have on themselves and on the pressure and velocity of the liquid flow. The analysis that follows incorporates these interactions using the equations for nonbarotropic homogeneous flow.