Rayleigh-Plesset Equation

Consider a spherical bubble of radius, $R(t)$ (where t is time), in an infinite domain of liquid whose temperature and pressure far from the bubble are T_∞ and $p_\infty(t)$ respectively. The temperature, T_∞, is assumed to be a simple constant since temperature gradients are not considered. On the other hand, the pressure, $p_\infty(t)$, is assumed to be a known (and perhaps controlled) input that regulates the growth or collapse of the bubble.

Though compressibility of the liquid can be important in the context of bubble collapse, it will, for the present, be assumed that the liquid density, ρ_L, is a constant. Furthermore, the dynamic viscosity, μ_L, is assumed constant and uniform. It will also be assumed that the contents of the bubble are homogeneous and that the temperature, $T_B(t)$, and pressure, $p_B(t)$, within the bubble are always uniform. These assumptions may not be justified in circumstances that will be identified as the analysis proceeds.

The radius of the bubble, $R(t)$, will be one of the primary results of the analysis. As indicated in figure 1, radial position within the liquid will be denoted by the distance, r, from the center of the bubble; the pressure, $p(r,t)$, radial outward velocity, $u(r,t)$, and temperature, $T(r,t)$, within the liquid will be so designated. Conservation of mass requires that

$$u(r,t) = \frac{F(t)}{r^2} \tag{Ngb1}$$

where $F(t)$ is related to $R(t)$ by a kinematic boundary condition at the bubble surface. In the idealized case of zero mass transport across this interface, it is clear that $u(R,t) = dR/dt$ and hence

$$F(t) = R^2\frac{dR}{dt} \tag{Ngb2}$$

This is often a good approximation even when evaporation or condensation is occurring at the interface (Brennen 1995) provided the vapor density is much smaller than the liquid density.

Assuming a Newtonian liquid, the Navier-Stokes equation for motion in the r direction,

$$-\frac{1}{\rho_L}\frac{\partial p}{\partial r} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \nu_L\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial u}{\partial r}) - \frac{2u}{r^2}\right\} \tag{Ngb3}$$

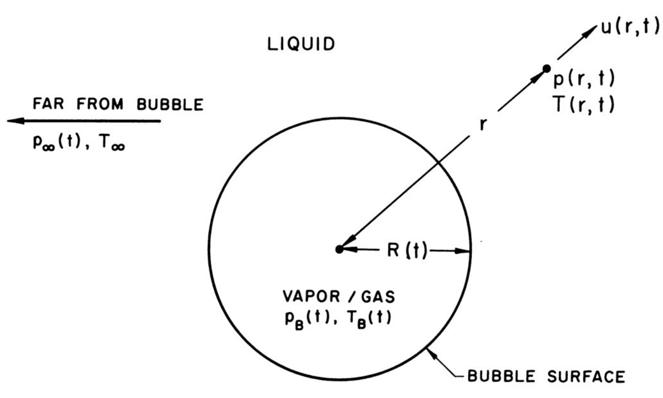

Figure 1: Schematic of a spherical bubble in an infinite liquid.

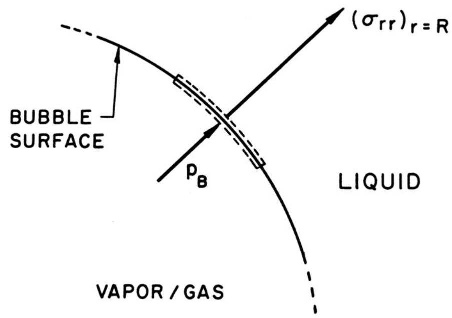

Figure 2: Portion of the spherical bubble surface.

yields, after substituting for u from $u = F(t)/r^2$:

$$-\frac{1}{\rho_L}\frac{\partial p}{\partial r} = \frac{1}{r^2}\frac{dF}{dt} - \frac{2F^2}{r^5} \tag{Ngb4}$$

Note that the viscous terms vanish; indeed, the only viscous contribution to the Rayleigh-Plesset equation (Ngb8) comes from the dynamic boundary condition at the bubble surface. Equation (Ngb4) can be integrated to give

$$\frac{p - p_\infty}{\rho_L} = \frac{1}{r}\frac{dF}{dt} - \frac{1}{2}\frac{F^2}{r^4} \tag{Ngb5}$$

after application of the condition $p \to p_\infty$ as $r \to \infty$.

To complete this part of the analysis, a dynamic boundary condition on the bubble surface must be constructed. For this purpose consider a control volume consisting of a small, infinitely thin lamina containing a segment of interface (figure 2). The net force on this lamina in the radially outward direction per unit area is

$$(\sigma_{rr})_{r=R} + p_B - \frac{2S}{R} \tag{Ngb6}$$

or, since $\sigma_{rr} = -p + 2\mu_L \partial u/\partial r$, the force per unit area is

$$p_B - (p)_{r=R} - \frac{4\mu_L}{R}\frac{dR}{dt} - \frac{2S}{R} \tag{Ngb7}$$

In the absence of mass transport across the boundary (evaporation or condensation) this force must be zero, and substitution of the value for $(p)_{r=R}$ from equation **??** with $F = R^2\ dR/dt$ yields the generalized Rayleigh-Plesset equation for bubble dynamics:

$$\frac{p_B(t) - p_\infty(t)}{\rho_L} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_L}{R}\frac{dR}{dt} + \frac{2S}{\rho_L R} \tag{Ngb8}$$

Given $p_\infty(t)$ this represents an equation that can be solved to find $R(t)$ provided $p_B(t)$ is known. In the absence of the surface tension and viscous terms, it was first derived and used by Rayleigh (1917). Plesset (1949) first applied the equation to the problem of traveling cavitation bubbles.