Nonlinear Effects

Due to the nonlinearities in the governing equations, particularly the Rayleigh-Plesset equation (Ngc2), the response of a bubble subjected to pressure oscillations will begin to exhibit important nonlinear effects as the amplitude of the oscillations is increased. In the last few sections of this chapter we briefly review some of these nonlinear effects. Much of the research appears in the context of acoustic cavitation, a subject with an extensive literature that is reviewed in detail elsewhere (Flynn 1964; Neppiras 1980; Plesset and Prosperetti 1977; Prosperetti 1982, 1984; Crum 1979; Young 1989). We include here a brief summary of the basic phenomena.

As the amplitude increases, the bubble *may* continue to oscillate stably. Such circumstances are referred to as *stable acoustic cavitation* to distinguish them from those of the *transient* regime described below. Several different nonlinear phenomena can affect stable acoustic cavitation in important ways. Among these are the production of subharmonics, the phenomenon of rectified diffusion (see section (Ngl)) and the generation of Bjerknes forces (see section (Nfe)). At larger amplitudes the change in bubble size during a single period of oscillation can become so large that the bubble undergoes a cycle of explosive cavitation growth and violent collapse similar to that described earlier in the chapter. Such a response is termed *transient acoustic cavitation* and is distinguished from stable acoustic cavitation by the fact that the bubble radius changes by several orders of magnitude during each cycle.

As Plesset and Prosperetti (1977) have detailed in their review of the subject, when a liquid that will inevitably contain microbubbles is irradiated with sound of a given frequency, ω, the nonlinear response results in harmonic dispersion, that not only produces harmonics with frequencies that are integer multiples of ω (superharmonics) but, more unusually, subharmonics with frequencies less than ω of the form $m\omega/n$ where m and n are integers. Both the superharmonics and subharmonics become more prominent as the amplitude of excitation is increased. The production of subharmonics was first observed experimentally by Esche (1952), and possible origins of this nonlinear effect were explored in detail by Noltingk and Neppiras (1950, 1951), Flynn (1964), Borotnikova and Soloukin (1964), and Neppiras (1969), among others. Lauterborn (1976) examined numerical solutions for a large number of different excitation frequencies and was able to demonstrate the progressive development of the peak responses at subharmonic frequencies as the amplitude of the excitation is increased. Nonlinear effects not only create these subharmonic peaks but also cause the resonant peaks to be shifted to lower frequencies, creating discontinuities that correspond to bifurcations in the solutions. The weakly nonlinear analysis of Brennen (1995) produces similar phenomena. In recent years, the modern methods of nonlinear dynamical systems analysis have been applied to this problem by Lauterborn and Suchla (1984), Smereka, Birnir, and Banerjee (1987), Parlitz *et al.* (1990), and others and have led to further understanding of the bifurcation diagrams and strange attractor maps that arise in the dynamics of single bubble oscillations.

Finally, we comment on the phenomenon of transient cavitation in which a phase of explosive cavitation growth and collapse occurs each cycle of the imposed pressure oscillation. We seek to establish the level of pressure oscillation at which this will occur, known as the threshold for transient cavitation (see Noltingk and Neppiras 1950, 1951, Flynn 1964, Young 1989). The answer depends on the relation between the radian frequency, ω, of the imposed oscillations and the natural frequency, ω_n, of the bubble. If $\omega \ll \omega_n$, then the liquid inertia is relatively unimportant in the bubble dynamics and the bubble will respond quasistatically. Under these circumstances the Blake criterion (see section (Nge), equation (Nge5)) will hold and the critical conditions will be reached when the minimum instantaneous pressure just reaches the critical Blake threshold pressure. On the other hand, if $\omega \gg \omega_n$, the issue will involve the dynamics of bubble growth since inertia will determine the size of the bubble perturbations. The details of this bubble dynamic problem have been addressed by Flynn (1964) and convenient guidelines are provided by Apfel (1981).