Kinematic Wave Analysis

Consider the most basic model of two-component pipe flow (components A and B) in which the relative motion is non-negligible. We shall assume a pipe of uniform cross-section. In the absence of phase change the continuity equations become

$$\frac{\partial \alpha_A}{\partial t} + \frac{\partial j_A}{\partial s} = 0 \quad ; \quad \frac{\partial \alpha_B}{\partial t} + \frac{\partial j_B}{\partial s} = 0 \tag{Nsb1}$$

For convenience we set $\alpha = \alpha_A = 1 - \alpha_B$. Then, using the standard notation of equations (Nrl1) to (Nrl4), we expand α, j_A and j_B in terms of their mean values (denoted by an overbar) and harmonic perturbations (denoted by the tilde) at a frequency ω in the form used in expressions (Nrl1). The solution for the mean flow is simply

$$\frac{d(\bar{j}_A + \bar{j}_B)}{ds} = \frac{d\bar{j}}{ds} = 0 \tag{Nsb2}$$

and therefore $\bar{j}$ is a constant. Moreover, the following equations for the perturbations emerge:

$$\frac{\partial \tilde{j}_A}{\partial s} + i\omega\tilde{\alpha} = 0 \quad ; \quad \frac{\partial \tilde{j}_B}{\partial s} - i\omega\tilde{\alpha} = 0 \tag{Nsb3}$$

Now consider the additional information that is necessary in order to determine the dispersion equation and therefore the different modes of wave propagation that can occur in this flow. First, we note that

$$j_A = \alpha j + j_{AB} \quad ; \quad j_B = (1-\alpha)j - j_{AB} \tag{Nsb4}$$

and it is convenient to replace the variables, j_A and j_B, by j, the total volumetric flux, and j_{AB}, the drift flux. Substituting these expressions into equations (Nsb3), we obtain

$$\frac{\partial \tilde{j}}{\partial s} = 0 \quad ; \quad \frac{\partial(\bar{j}\tilde{\alpha} + \tilde{j}_{AB})}{\partial s} + i\omega\tilde{\alpha} = 0 \tag{Nsb5}$$

The first of these yields a uniform and constant value of $\tilde{j}$ that corresponds to a synchronous motion in which the entire length of the multiphase flow in the pipe is oscillating back and forth in unison. Such motion is not of interest here and we shall assume for the purposes of the present analysis that $\tilde{j} = 0$.

The second equation (Nsb5) has more interesting implications. It represents the connection between the two remaining fluctuating quantities, $\tilde{j}_{AB}$ and $\tilde{\alpha}$. To proceed further it is therefore necessary to find a second relation connecting these same quantities. It now becomes clear that, from a mathematical point of view, there is considerable simplicity in the the *Drift Flux Model* (sections (Nq)), in which it is assumed that the relative motion is governed by a simple algebraic relation connecting j_{AB} and α, We shall utilize that model here and assume the existence of a known, functional relation, $j_{AB}(\alpha)$. Then the second equation (Nsb5) can be written as

$$\left(\bar{j} + \left.\frac{dj_{AB}}{d\alpha}\right|_{\bar{\alpha}} \right) \frac{\partial \tilde{\alpha}}{\partial s} + i\omega\tilde{\alpha} = 0 \tag{Nsb6}$$

where $dj_{AB}/d\alpha$ is evaluated at $\alpha = \bar{\alpha}$ and is therefore a known function of $\bar{\alpha}$. It follows that the dispersion relation yields a single wave type given by the wavenumber, κ, and wave velocity, c, where

$$\kappa = -\frac{\omega}{\bar{j} + \left.\frac{dj_{AB}}{d\alpha}\right|_{\bar{\alpha}}} \qquad \text{and} \qquad c = \bar{j} + \left.\frac{dj_{AB}}{d\alpha}\right|_{\bar{\alpha}} \tag{Nsb7}$$

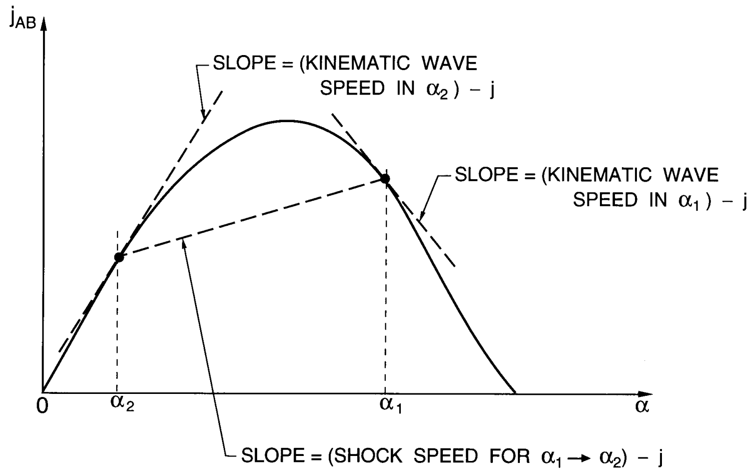

Figure 1: Kinematic wave speeds and shock speeds in a drift flux chart.

This is called a kinematic wave since its primary characteristic is the perturbation in the volume fraction and it travels at a velocity close to the velocity of the components. Indeed, in the absence of relative motion $c \rightarrow \bar{j} = u_A = u_B$.

The expression (Nsb7) (and the later expression (Nse5) for the kinematic shock speed) reveal that the propagation speed of kinematic waves (and shocks) relative to the total volumetric flux, j, can be conveniently displayed in a drift flux chart as illustrated in figure 1. The kinematic wave speed at a given volume fraction is the slope of the tangent to the drift flux curve at that point (plus j). This allows a graphical and comparative display of wave speeds that, as we shall demonstrate, is very convenient in flows that can be modeled using the drift flux methodology.