Multiphase Flow Notation

The notation that will be used is close to the standard described by Wallis (1969). It has however been slightly modified to permit more ready adoption to the Cartesian tensor form. In particular the subscripts that can be attached to a property will consist of a group of uppercase subscripts followed by lowercase subscripts. The lower case subscripts (i, ij, etc.) are used in the conventional manner to denote vector or tensor components. A single uppercase subscript (N) will refer to the property of a specific phase or component. In some contexts generic subscripts $N = A, B$ will be used for generality. However, other letters such as $N = C$ (continuous phase), $N = D$ (disperse phase), $N = L$ (liquid), $N = G$ (gas), $N = V$ (vapor) or $N = S$ (solid) will be used for clarity in other contexts. Finally two uppercase subscripts will imply the difference between the two properties for the two single uppercase subscripts.

Specific properties frequently used are as follows. *Volumetric fluxes* (volume flow per unit area) of individual components will be denoted by j_{Ai}, j_{Bi} ($i = 1$, 2 or 3 in three dimensional flow). These are sometimes referred to as superficial component velocities. The *total volumetric flux*, j_i is then given by

$$j_i = j_{Ai} + j_{Bi} + \cdots = \sum_N j_{Ni} \tag{Nac1}$$

Mass fluxes are similarly denoted by G_{Ai}, G_{Bi} or G_i. Thus if the densities of individual components are denoted by ρ_A, ρ_B it follows that

$$G_{Ai} = \rho_A j_{Ai} \ ; \ G_{Bi} = \rho_B j_{Bi} \ ; \ G_i = \sum_N \rho_N j_{Ni} \tag{Nac2}$$

Velocities of the specific phases are denoted by u_{Ai}, u_{Bi} or, in general, by u_{Ni}. The relative velocity between the two phases A and B will be denoted by u_{ABi} such that

$$u_{Ai} - u_{Bi} = u_{ABi} \tag{Nac3}$$

The volume fraction of a component or phase is denoted by α_N and, in the case of two components or phases, A and B, it follows that $\alpha_B = 1 - \alpha_A$. Though this is clearly a well defined property for any finite volume in the flow, there are some substantial problems associated with assigning a value to an infinitesimal volume or point in the flow. Provided these can be resolved, it follows that the volumetric flux of a component, N, and its velocity are related by

$$j_{Ni} = \alpha_N u_{Ni} \tag{Nac4}$$

and that

$$j_i = \alpha_A u_{Ai} + \alpha_B u_{Bi} + \cdots = \sum_N \alpha_N u_{Ni} \tag{Nac5}$$

Two other fractional properties are only relevant in the context of one-dimensional flows. The *volumetric quality*, β_N, is the ratio of the volumetric flux of the component, N, to the total volumetric flux, i.e.

$$\beta_N = j_N / j \tag{Nac6}$$

where the index i has been dropped from j_N and j because β is only used in the context of one-dimensional flows and the j_N, j refer to cross-sectionally averaged quantities.

The *mass fraction*, x_A, of a phase or component, A, is simply given by $\rho_A \alpha_A / \rho$ (see equation (Nac8) for ρ). On the other hand the *mass quality*, $\mathcal{X}_A$, is often referred to simply as *the quality* and is the ratio of the mass flux of component, A, to the total mass flux, or

$$\mathcal{X}_A = \frac{G_A}{G} = \frac{\rho_A j_A}{\sum\limits_N \rho_N j_N} \tag{Nac7}$$

Furthermore, when only two components or phases are present it is often redundant to use subscripts on the volume fraction and the qualities since $\alpha_A = 1 - \alpha_B$, $\beta_A = 1 - \beta_B$ and $\mathcal{X}_A = 1 - \mathcal{X}_B$. Thus unsubscripted quantities α, β and $\mathcal{X}$ will often be used in these circumstances.

It is clear that a multiphase mixture has certain *mixture* properties of which the most readily evaluated is the *mixture* density denoted by ρ and given by

$$\rho = \sum_N \alpha_N \rho_N \tag{Nac8}$$

On the other hand the specific enthalpy, h, and specific entropy, s, being defined as per unit mass rather than per unit volume are weighted according to

$$\rho h = \sum_N \rho_N \alpha_N h_N \; ; \; \rho s = \sum_N \rho_N \alpha_N s_N \tag{Nac9}$$

Other properties such as the *mixture* viscosity or thermal conductivity cannot be reliably obtained from such simple weighted means.

Aside from the relative velocities between phases that were described earlier, there are two other measures of relative motion that are frequently used. The *drift velocity* of a component is defined as the velocity of that component in a frame of reference moving at a velocity equal to the total volumetric flux, j_i, and is therefore given by, u_{NJi}, where

$$u_{NJi} = u_{Ni} - j_i \tag{Nac10}$$

Even more frequent use will be made of the *drift flux* of a component which is defined as the volumetric flux of a component in the frame of reference moving at j_i. Denoted by j_{NJi} this is given by

$$j_{NJi} = j_{Ni} - \alpha_N j_i = \alpha_N \left(u_{Ni} - j_i \right) = \alpha_N u_{NJi} \tag{Nac11}$$

It is particularly important to notice that the sum of all the drift fluxes must be zero since from equation (Nac11)

$$\sum_N j_{NJi} = \sum_N j_{Ni} - j_i \sum_N \alpha_N = j_i - j_i = 0 \tag{Nac12}$$

When only two phases or components, A and B, are present it follows that $j_{AJi} = -j_{BJi}$ and hence it is convenient to denote both of these drift fluxes by the vector j_{ABi} where

$$j_{ABi} = j_{AJi} = -j_{BJi} \tag{Nac13}$$

Moreover it follows from (Nac11) that

$$j_{ABi} = \alpha_A \alpha_B u_{ABi} = \alpha_A (1 - \alpha_A) u_{ABi} \tag{Nac14}$$

and hence the drift flux, j_{ABi} and the relative velocity, u_{ABi}, are simply related.

Finally, it is clear that certain basic relations follow from the above definitions and it is convenient to identify these here for later use. First the relations between the volume and mass qualities that follow from equations (Nac6) and (Nac7) only involve ratios of the densities of the components:

$$\mathcal{X}_A = \beta_A / \sum_N \left(\frac{\rho_N}{\rho_A} \right) \beta_N \quad ; \quad \beta_A = \mathcal{X}_A / \sum_N \left(\frac{\rho_A}{\rho_N} \right) \mathcal{X}_N \tag{Nac15}$$

On the other hand the relation between the volume fraction and the volume quality necessarily involves some measure of the relative motion between the phases (or components). The following useful results for two-phase (or two-component) one-dimensional flows can readily be obtained from (Nac11) and (Nac6)

$$\beta_N = \alpha_N + \frac{j_{NJ}}{j} \; ; \; \beta_A = \alpha_A + \frac{j_{AB}}{j} \; ; \; \beta_B = \alpha_B - \frac{j_{AB}}{j} \tag{Nac16}$$

which demonstrate the importance of the drift flux as a measure of the relative motion.