Flow Regime Rheologies

Campbell (2002, 2003) has carried out an extensive series of computer simulations of shear flows designed to identify further characteristics of the flow regimes and, in particular, to identify the boundaries between them. Though his results are complicated because the simulations carried out with the solids fraction fixed seem to exhibit differences from those carried out with the normal stress or overburden fixed, we give here a brief overview of a few key features and results emerging from the fixed normal stress simulations. As one might expect, the flows at low values of $K/\rho_S D^3\dot{\gamma}^2$ are dominated by force chains that carry most of the shear stress in the shear flow. These chains form, rotate and disperse continually during shear (Drescher and De Josselin de Jong 1972, Cundall and Strack 1979). Evaluating the typical particle contact time, Campbell finds that, in this elastic-quasistatic regime the dynamics are not correlated with the binary contact time but are determined by the shear rate. This clearly indicates multiple particle structures (force chains) whose lifetime is determined by their rotation under shear. However, as $K/\rho_S D^3\dot{\gamma}^2$ is increased and the flow approaches the rapid granular flow limit, the typical contact time asymptotes to the binary contact time indicating the dominance of simple binary collisions and the disappearance of force chains.

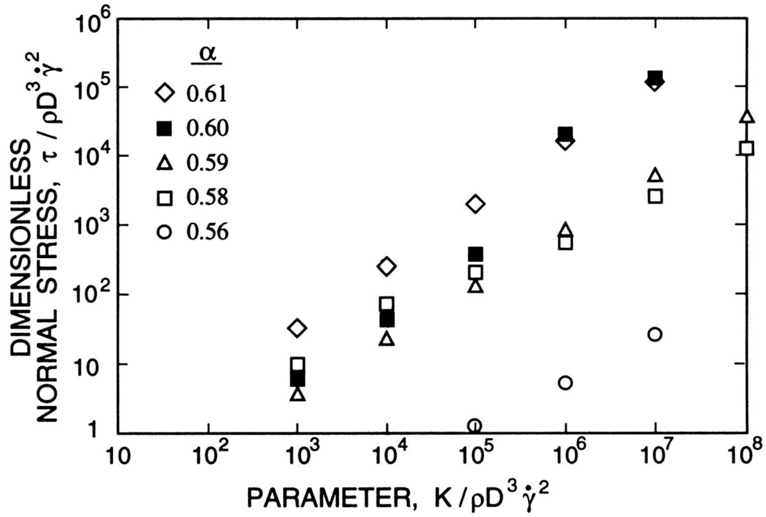

Figure 1: Typical non-dimensional stress, $\tau/\rho_S D^2\dot{\gamma}^2$ (in this case a normal stress) in a uniform shear flow as a function of the parameter, $K/\rho_S D^3\dot{\gamma}^2$, for various solids fractions, α, a friction coefficient $\mu^* = 0.5$ and a coefficient of restitution of $\epsilon = 0.7$ (adapted from Campbell 2003).

Figure 1 is a typical result from Campbell's simulations at fixed normal stress and plots the dimensionless stress $\tau/\rho_S D^2\dot{\gamma}^2$ against the parameter $K/\rho_S D^3\dot{\gamma}^2$ for various values of the solids fraction, α. Note that at high solids fractions the slopes of the curves approach unity indicating that the ratio, $\tau D/K$, is constant in that part of the parameter space. This is therefore the elastic-quasistatic regime. At lower solids fractions, the dimensionless stress is a more complex function of both solids fraction and the parameter, $K/\rho_S D^3\dot{\gamma}^2$, thus indicating the appearance of inertial effects. Another interesting feature is the ratio of the shear to normal stress, τ_s/τ_n, and the manner in which it changes with the change in flow regime. At high $K/\rho_S D^3\dot{\gamma}^2$ this ratio asymptotes to a constant value that corresponds to the internal friction angle used in soil mechanics (and is closely related to the interparticle friction coefficient, μ^*). However, as $K/\rho_S D^3\dot{\gamma}^2$ is decreased (at constant normal stress) the simulations show τ_s/τ_n increasing with the increases being greater the smaller the normal stress.

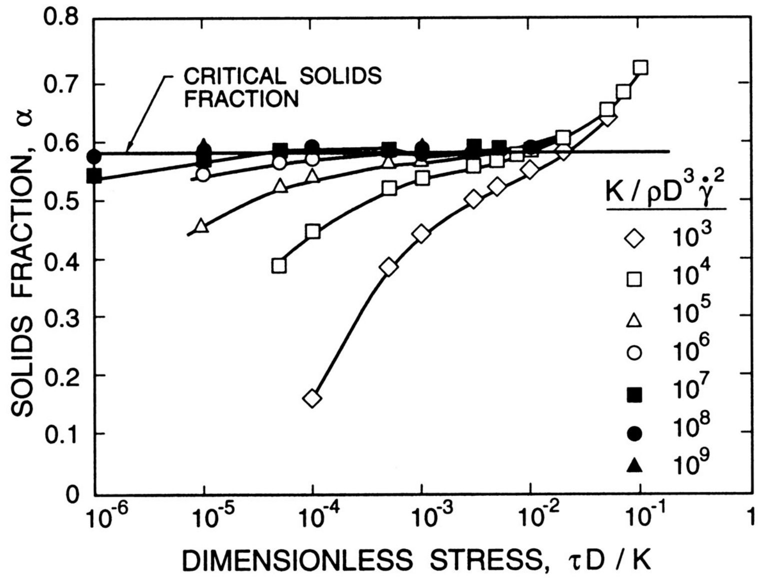

Figure 2: The variation of the solids fraction, α, with the dimensionless applied stress, $\tau D/K$, in a uniform shear flow with fixed normal stress for various values of the parameter, $K/\rho_S D^3\dot{\gamma}^2$. Computer simulation data from Campbell (2003) for the case of a friction coefficient of $\mu^* = 0.5$ and a coefficient of restitution of $\epsilon = 0.7$.

Fundamental rheological information such as given in figure 1 can be used to construct granular flow regime maps. However, it is first necessary to discuss the solids fraction, α, and how that is established in most granular flows. The above analysis assumed, for convenience, that α was known and sought expressions for the stresses, τ, both normal and tangential. In practical granular flows, the normal stress or overburden is usually established by the circumstances of the flow and by the gravitational forces acting on the material. The solids fraction results from the rheology of the flow. Under such circumstances, the data required is the solids fraction, α as a function of the dimensionless overburden, $\tau D/K$ for various values of the parameter, $K/\rho_S D^3\dot{\gamma}^2$. An example from Campbell (2003), is shown in figure 2 and illustrates another important feature of granular dynamics. At high values of the overburden and solids fraction, the rate parameter, $K/\rho_S D^3\dot{\gamma}^2$ plays little role and the solids fraction simply increases with the overburden. As the solids fraction decreases in order to facilitate flow, then, for low shear rates or high values of $K/\rho_S D^3\dot{\gamma}^2$, the material asymptotes to a *critical* solids fraction of about 0.59 in the case of figure 2. This is the critical state phenomenon familiar to soil mechanicists (see, for example, Schofield and Wroth 1968). However, at higher shear rates, lower values of $K/\rho_S D^3\dot{\gamma}^2$, and lower overburdens, the material expands below the critical solids fraction as the material moves into the inertial regime and the collisions and interactions between the particles cause the material to expand. Figure 2 therefore displays both the traditional soil mechanics behavior and the classic kinetic theory behavior that results from the dominance of random, collisional motions. We also see that the traditional critical solids fraction could be considered as the dividing line between the inertial and elastic-quasistatic regimes of flow.