Stratified Flow Instability

As a first example, consider the stability of the horizontal stratified flow depicted in figure 1 where the destabilizing Bernoulli effect is primarily opposed by a stabilizing buoyancy force. An approximate instability condition is readily derived by observing that the formation of a wave (such as that depicted in figure 1) will lead to a reduced pressure, p_A, in the gas in the orifice formed by that wave. The reduction below the mean gas pressure, $\bar{p}_G$, will be given by Bernoulli's equation as

$$p_A - \bar{p}_G = -\rho_G u_G^2 a/h \tag{Njp1}$$

provided $a \ll h$. The restraining pressure is given by the buoyancy effect of the elevated interface, namely $(\rho_L - \rho_G)ga$. It follows that the flow will become unstable when

$$u_G^2 > gh\Delta\rho/\rho_G \tag{Njp2}$$

In this case the liquid velocity has been neglected since it is normally small compared with the gas velocity. Consequently, the instability criterion provides an upper limit on the gas velocity that is, in effect, the velocity difference. Taitel and Dukler (1976) compared this prediction for the boundary of the stratified flow regime in a horizontal pipe of diameter, d, with the experimental observations of Mandhane *et al.* (1974) and found substantial agreement. This can be demonstrated by observing that, from equation (Njp2),

$$j_G = \alpha u_G = C(\alpha)\alpha(gd\Delta\rho/\rho_G)^{\frac{1}{2}} \tag{Njp3}$$

where $C(\alpha) = (h/d)^{\frac{1}{2}}$ is some simple monotonically increasing function of α that depends on the pipe cross-section. For example, for the $2.5cm$ pipe of figure 3, section (Njb), the factor $(gd\Delta\rho/\rho_G)^{\frac{1}{2}}$ in equation (Njp3) will have a value of approximately $15m/s$. As can be observed in figure 3, section (Njb), this is in close agreement with the value of j_G at which the flow at low j_L departs from the stratified regime and begins to become wavy and then annular. Moreover the factor $C(\alpha)\alpha$ should decrease as j_L increases and, in figure 3, section (Njb), the boundary between stratified flow and wavy flow also exhibits this decrease.

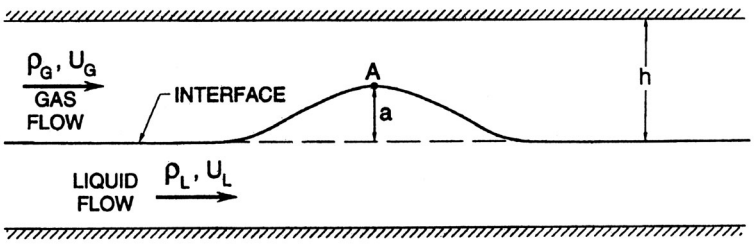

Figure 1: Sketch showing the notation for stratified flow instability.