Stability of Disperse Mixtures

It transpires that a homogeneous, quiescent multiphase mixture may be internally unstable as a result of gravitationally-induced relative motion. This instability was first described for fluidized beds by Jackson (1963). It results in horizontally-oriented, vertically-propagating volume fraction waves or layers of the disperse phase. To evaluate the stability of a uniformly dispersed two component mixture with uniform relative velocity induced by gravity and a density difference, Jackson constructed a model consisting of the following system of equations:

1. The number continuity equation (Nbc1) for the particles (density, ρ_D, and volume fraction, $\alpha_D = \alpha$) is:
$$\frac{\partial \alpha}{\partial t} + \frac{\partial(\alpha u_D)}{\partial y} = 0 \quad \text{(Njm1)}$$
where all velocities are in the vertically upward direction.

2. Volume continuity for the suspending fluid (assuming constant density, ρ_C, and zero mass interaction, $\mathcal{I}_N = 0$)
$$\frac{\partial \alpha}{\partial t} - \frac{\partial((1-\alpha)u_C)}{\partial y} = 0 \quad \text{(Njm2)}$$

3. Individual phase momentum equations (Nbe5) for both the particles and the fluid assuming constant densities and no deviatoric stress:
$$\rho_D \alpha \left\{ \frac{\partial u_D}{\partial t} + u_D \frac{\partial u_D}{\partial y} \right\} = -\alpha \rho_D g + \mathcal{F}_D \quad \text{(Njm3)}$$
$$\rho_C (1-\alpha) \left\{ \frac{\partial u_C}{\partial t} + u_C \frac{\partial u_C}{\partial y} \right\} = -(1-\alpha)\rho_C g - \frac{\partial p}{\partial y} - \mathcal{F}_D \quad \text{(Njm4)}$$

4. A force interaction term of the form given by equation (Nbe7). Jackson constructs a component, $\mathcal{F}'_{Dk}$, due to the relative motion of the form
$$\mathcal{F}'_D = q(\alpha)(1-\alpha)(u_C - u_D) \quad \text{(Njm5)}$$
where q is assumed to be some function of α. Note that this is consistent with a low Reynolds number flow.

Jackson then considered solutions of these equations that involve small, linear perturbations or waves in an otherwise homogeneous mixture. Thus the flow was decomposed into:

1. A uniform, homogeneous fluidized bed in which the mean values of u_D and u_C are respectively zero and some adjustable constant. To maintain generality, we will characterize the relative motion by the drift flux, $j_{CD} = \alpha(1-\alpha)u_C$.

2. An unsteady linear perturbation in the velocities, pressure and volume fraction of the form $exp\{i\kappa y + (\zeta - i\omega)t\}$ that models waves of wavenumber, κ, and frequency, ω, traveling in the y direction with velocity ω/κ and increasing in amplitude at a rate given by ζ.

Substituting this decomposition into the system of equations described above yields the following expression for $(\zeta - i\omega)$:

$$(\zeta - i\omega)\frac{j_{CD}}{g} = \pm K_2\{1 + 4iK_3 + 4K_1K_3^2 - 4iK_3(1+K_1)K_4\}^{\frac{1}{2}} - K_2(1+2iK_3) \tag{Njm6}$$

where the constants K_1 through K_3 are given by

$$K_1 = \frac{\rho_D}{\rho_C}\frac{(1-\alpha)}{\alpha} \quad ; \quad K_2 = \frac{(\rho_D - \rho_C)\alpha(1-\alpha)}{2\{\rho_D(1-\alpha)+\rho_C\alpha\}}$$

$$K_3 = \frac{\kappa\, j_{CD}^2}{g\alpha(1-\alpha)^2\{\rho_D/\rho_C - 1\}} \tag{Njm7}$$

and K_4 is given by

$$K_4 = 2\alpha - 1 + \frac{\alpha(1-\alpha)}{q}\frac{dq}{d\alpha} \tag{Njm8}$$

It transpires that K_4 is a critical parameter in determining the stability and it, in turn, depends on how q, the factor of proportionality in equation (Njm5), varies with α. Here we examine two possible functions, $q(\alpha)$. The Carman-Kozeny equation (Nel4) for the pressure drop through a packed bed is appropriate for slow viscous flow and leads to $q \propto \alpha^2/(1-\alpha)^2$; from equation (Njm8) this yields $K_4 = 2\alpha + 1$ and is an example of low Reynolds number flow. As a representative example of higher Reynolds number flow we take the relation (Nel8) due to Wallis (1969) and this leads to $q \propto \alpha/(1-\alpha)^{b-1}$ (recall Wallis suggests $b = 3$); this yields $K_4 = b\alpha$. We will examine both of these examples of the form of $q(\alpha)$.

Note that the solution (Njm6) yields the non-dimensional frequency and growth rate of waves with wavenumber, κ, as functions of just three dimensionless variables, the volume fraction, α, the density ratio, ρ_D/ρ_C, and the relative motion parameter, $j_{CD}/(g/\kappa)^{\frac{1}{2}}$, similar to a Froude number. Note also that equation (Njm6) yields two roots for the dimensionless frequency, $\omega j_{CD}/g$, and growth rate, $\zeta j_{CD}/g$. Jackson demonstrates that the negative sign choice is an attenuated wave; consequently we focus exclusively on the positive sign choice that represents a wave that propagates in the direction of the drift flux, j_{CD}, and grows exponentially with time. It is also easy to see that the growth rate tends to infinity as $\kappa \rightarrow \infty$. However, it is meaningless to consider wavelengths less than the inter-particle distance and therefore the focus should be on waves of this order since they will predominate. Therefore, in the discussion below, it is assumed that the κ^{-1} values of primary interest are of the order of the typical inter-particle distance.

Figure 1 presents typical dimensionless growth rates for various values of the parameters α, ρ_D/ρ_C, and $j_{CD}/(g/\kappa)^{\frac{1}{2}}$ for both the Carman-Kozeny and Wallis expressions for K_4. In all cases the growth rate increases with the wavenumber κ, confirming the fact that the fastest growing wavelength is the smallest that is relevant. We note, however, that a more complete linear analysis by Anderson and Jackson (1968) (see also Homsy *et al.* 1980, Jackson 1985, Kytömaa 1987) that includes viscous effects yields a wavelength that has a maximum growth rate. Figure 1 also demonstrates that the effect of void fraction is modest; though the lines for $\alpha = 0.5$ lie below those for $\alpha = 0.1$ this must be weighed in conjunction with the fact that the interparticle distance is greater in the latter case. Gas and liquid fluidized beds are typified by ρ_D/ρ_C values of 3000 and 3 respectively; since the lines for these two cases are not far apart, the primary difference is the much larger values of j_{CD} in gas-fluidized beds. Everything else being equal, increasing j_{CD} means following a line of slope 1 in figure 1 and this implies much larger values of the growth rate in gas-fluidized beds. This is in accord with the experimental observations.

As a postscript, it must be noted that the above analysis leaves out many effects that may be consequential. As previously mentioned, the inclusion of viscous effects is important at least for lower Reynolds number flows. At higher particle Reynolds numbers, even more complex interactions can occur as particles encounter the wakes of other particles. For example, Fortes *et al.* (1987) demonstrated the complexity of particle-particle interactions under those circumstances and Joseph (1993) provides a summary of how

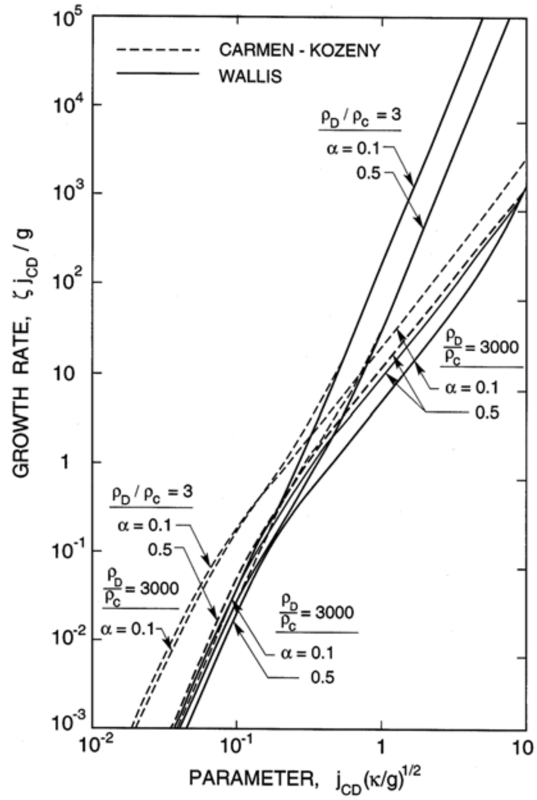

Figure 1: The dimensionless growth rate $\zeta j_{CD}/g$ plotted against the parameter $j_{CD}/(g/\kappa)^{\frac{1}{2}}$ for various values of α and ρ_D/ρ_C and for both $K_4 = 2\alpha + 1$ and $K_4 = 3\alpha$.

the inhomogeneities or volume fraction waves evolve with such interactions. General analyses of kinematic waves are contained in sections (Ns) and the reader is referred to that section for details.