Thermally Controlled Growth

When the first critical time is exceeded it is clear that the relative importance of the various terms in the Rayleigh-Plesset equation, (Ngc2), will change. The most important terms become the driving term (1) and the thermal term (2) whose magnitude is much larger than that of the inertial terms (4). Hence if the tension $(p_V - p_\infty^*)$ remains constant, then the solution using the form of equation (Ngc14) for the thermal term must have $n = \frac{1}{2}$ and the asymptotic behavior is

$$R = \frac{(p_V - p_\infty^*)t^{\frac{1}{2}}}{\rho_L \Sigma(T_\infty) C(\frac{1}{2})} \quad \underline{\text{or}} \quad n = \frac{1}{2} \quad ; \quad R^* = \frac{(p_V - p_\infty^*)}{\rho_L \Sigma(T_\infty) C(\frac{1}{2})} \tag{Ngg1}$$

Consequently, as time proceeds, the inertial, viscous, gaseous, and surface tension terms in the Rayleigh-Plesset equation all rapidly decline in importance. In terms of the superheat, ΔT, rather than the tension

$$R = \frac{1}{2C(\frac{1}{2})} \frac{\rho_L c_{PL} \Delta T}{\rho_V \mathcal{L}} (\mathcal{D}_L t)^{\frac{1}{2}} \tag{Ngg2}$$

where the group $\rho_L c_{PL} \Delta T / \rho_V \mathcal{L}$ is termed the Jakob Number in the context of pool boiling and $\Delta T = T_w - T_\infty$, T_w being the wall temperature. We note here that this section will address only the issues associated with bubble growth in the liquid bulk. The presence of a nearby wall (as is the case in most boiling) causes details and complications the discussion of which is delayed until sections (Ni).

The result, equation (Ngg1) or (Ngg2), demonstrates that the rate of growth of the bubble decreases substantially after the first critical time, t_{c1}, is reached and that R subsequently increases like $t^{\frac{1}{2}}$ instead of t. Moreover, since the thermal boundary layer also increases like $(\mathcal{D}_L t)^{\frac{1}{2}}$, the Plesset-Zwick assumption remains valid indefinitely. An example of this thermally inhibited bubble growth is including in figure 1, which is taken from Dergarabedian (1953). We observe that the experimental data and calculations using the Plesset-Zwick method agree quite well.

When bubble growth is caused by decompression so that $p_\infty(t)$ changes substantially with time during growth, the simple approximate solution of equation (Ngg1) no longer holds and the analysis of the unsteady thermal boundary layer surrounding the bubble becomes considerably more complex. One must then solve

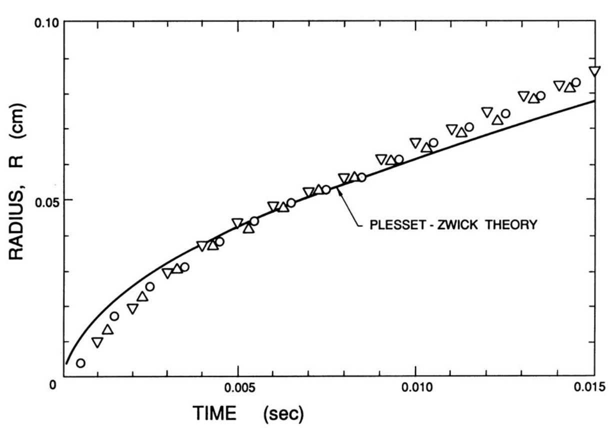

Figure 1: Experimental observations of the growth of three vapor bubbles (○, △, ▽) in superheated water at 103.1°C compared with the growth expected using the Plesset-Zwick theory (adapted from Dergarabedian 1953).

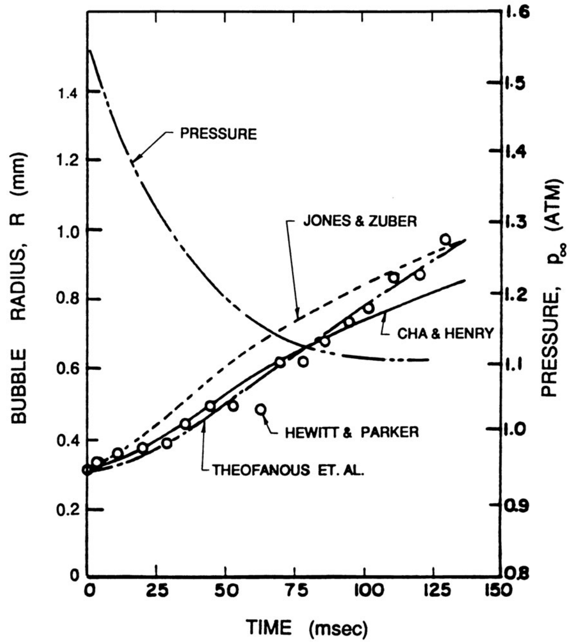

Figure 2: Data from Hewitt and Parker (1968) on the growth of a vapor bubble in liquid nitrogen (pressure/time history also shown) and comparison with the analytical treatments by Theofanous *et al.* (1969), Jones and Zuber (1978), and Cha and Henry (1981).

the diffusion equation (Ngc5), the energy equation (usually in the approximate form of equation (Ngc7)) and the Rayleigh-Plesset equation (Ngc2) simultaneously, though for the thermally controlled growth being considered here, most of the terms in equation (Ngc2) become negligible so that the simplification, $p_V(T_B) = p_\infty(t)$, is usually justified. When p_∞ is a constant this reduces to the problem treated by Plesset and Zwick (1952) and later addressed by Forster and Zuber (1954) and Scriven (1959). Several different approximate solutions to the general problem of thermally controlled bubble growth during liquid decompression have been put forward by Theofanous *et al.* (1969), Jones and Zuber (1978) and Cha and Henry (1981). All three analyses yield qualitatively similar results that also agree quite well with the experimental data of Hewitt and Parker (1968) for bubble growth in liquid nitrogen. Figure 2 presents a typical example of the data of Hewitt and Parker and a comparison with the three analytical treatments mentioned above.

Several other factors can complicate and alter the dynamics of thermally controlled growth. Nonequilibrium effects (Schrage 1953) can occur at very high evaporation rates where the liquid at the interface is no longer in thermal equilibrium with the vapor in the bubble and these have been explored by Theofanous *et al.* (1969) and Plesset and Prosperetti (1977) among others. The consensus seems to be that this effect is insignificant except, perhaps, in some extreme circumstances. There is no clear indication in the experiments of any appreciable departure from equilibrium.

More important are the modifications to the heat transfer mechanisms at the bubble surface that may be caused by surface instabilities or by convective heat transfer. These are reviewed in Brennen (1995). Shepherd and Sturtevant (1982) and Frost and Sturtevant (1986) have examined rapidly growing nucleation bubbles near the limit of superheat and have found growth rates substantially larger than expected when the bubble was in the thermally controlled growth phase. Photographs (see figure 3) reveal that the surfaces of those particular bubbles are rough and irregular. The enhancement of the heat transfer caused by this roughening is probably responsible for the larger than expected growth rates. Shepherd and Sturtevant (1982) attribute the roughness to the development of a baroclinic interfacial instability similar

Figure 3: Typical photographs of a rapidly growing bubble in a droplet of superheated ether suspended in glycerine. The bubble is the dark, rough mass; the droplet is clear and transparent. The photographs, which are of different events, were taken 31, 44, and 58 μs after nucleation and the droplets are approximately $2mm$ in diameter. Reproduced from Frost and Sturtevant (1986) with the permission of the authors.

to the Landau-Darrieus instablity of flame fronts. In other circumstances, Rayleigh-Taylor instability of the interface could give rise to a similar effect (Reynolds and Berthoud 1981).