Thermal Effects on Growth

In sections (Ngd) through (Nge) some of the characteristics of bubble dynamics in the absence of thermal effects were explored. It is now necessary to examine the regime of validity of those analyses. First we evaluate the magnitude of the thermal term (2) in equation (Ngc4) (see also equation (Ngc14)) that was neglected in order to produce equation (Ngd2).

First examine the case of bubble growth. The asymptotic growth rate given by equation (Ngd8) is constant and hence in the characteristic case of a constant p_∞, terms (1), (3), (4), (5), and (6) in equation (Ngc2) are all either constant or diminishing in magnitude as time progresses. Note that a constant, asymptotic growth rate corresponds to the case

$$n = 1 \quad ; \quad R^* = \{2(p_V - p_\infty^*)/3\rho_L\}^{\frac{1}{2}} \tag{Ngf1}$$

in equation (Ngc11). Consequently, according to equation (Ngc14), the thermal term (2) in its linearized form for small $(T_\infty - T_B)$ will be given by

$$\text{term}(2) = \Sigma(T_\infty)C(1)R^* t^{\frac{1}{2}} \tag{Ngf2}$$

Under these conditions, even if the thermal term is initially negligible, it will gain in magnitude relative to all the other terms and will ultimately affect the growth in a major way. Parenthetically it should be added that the Plesset-Zwick assumption of a small thermal boundary layer thickness, δ_T, relative to R can be shown to hold throughout the inertially controlled growth period since δ_T increases like $(\mathcal{D}_L t)^{\frac{1}{2}}$ whereas R is increasing linearly with t. Only under circumstances of very slow growth might the assumption be violated.

Using the relation (Ngf2), one can therefore define a critical time, t_{c1} (called the first critical time), during growth when the order of magnitude of term (2) in equation (Ngc2) becomes equal to the order of magnitude of the retained terms, as represented by $(dR/dt)^2$. This first critical time is given by

$$t_{c1} = \frac{(p_V - p_\infty^*)}{\rho_L} \cdot \frac{1}{\Sigma^2} \tag{Ngf3}$$

where the constants of order unity have been omitted for clarity. Thus t_{c1} depends not only on the tension $(p_V - p_\infty^*)/\rho_L$ but also on $\Sigma(T_\infty)$, a purely thermophysical quantity that is a function only of the liquid temperature. Recalling equation (Ngc15),

$$\Sigma(T) = \frac{\mathcal{L}^2 \rho_V^2}{\rho_L^2 c_{PL} T_\infty \mathcal{D}_L^{\frac{1}{2}}} \tag{Ngf4}$$

it can be anticipated that Σ^2 will change by many, many orders of magnitude in a given liquid as the temperature T_∞ is varied from the triple point to the critical point since Σ^2 is proportional to $(\rho_V/\rho_L)^4$. As a result the critical time, t_{c1}, will vary by many orders of magnitude. Some values of Σ for a number of liquids are plotted in figure 1 as a function of the reduced temperature T/T_C. As an example, consider a typical cavitating flow experiment in a water tunnel with a tension of the order of $10^4\ kg/m\ s^2$. Since water at $20°C$ has a value of Σ of about $1\ m/s^{\frac{3}{2}}$, the first critical time is of the order of $10s$, which is very much longer than the time of growth of bubbles. Hence the bubble growth occurring in this case is unhindered by thermal effects; it is *inertially controlled* growth. If, on the other hand, the tunnel water were heated to $100°C$ or, equivalently, one observed bubble growth in a pot of boiling water at superheat of $2°K$, then since $\Sigma \approx 10^3\ m/s^{\frac{3}{2}}$ at $100°C$ the first critical time would be $10\mu s$. Thus virtually all the bubble growth observed would be *thermally controlled.*

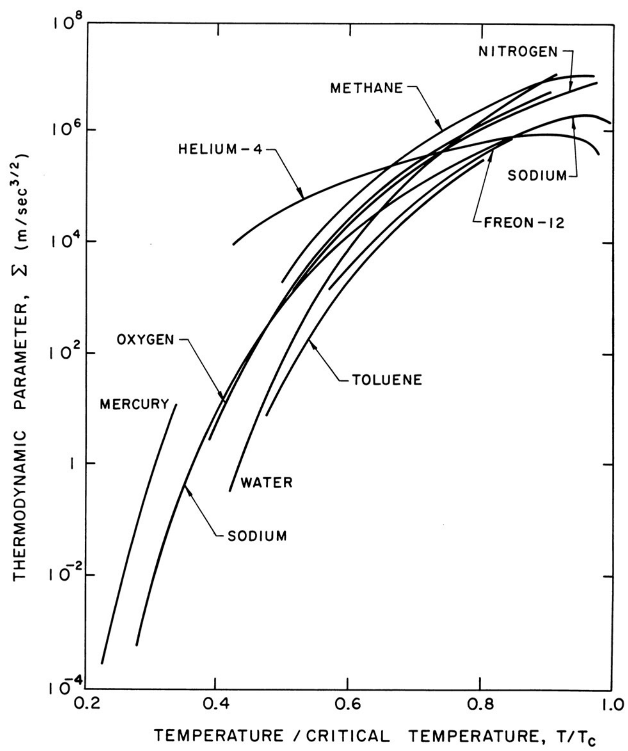

Figure 1: Values of the thermodynamic parameter, Σ, for various saturated liquids as a function of the reduced temperature, T/T_C.