Solution to Problem 505A

From the potential flow around a sphere we know that the velocity on the surface of the sphere varies with angular position, θ, like $(3/2)U\sin\theta$ where U is the velocity of the oncoming uniform stream and θ is the angle measured from the front stagnation point. The point A has $\theta = \beta + \alpha$ while the point B has $\theta = \beta - \alpha$. Denoting the magnitude of the velocities at these two points by u_A and u_B it follows that

$$u_A = \frac{3U}{2}\sin(\beta+\alpha) \quad ; \quad u_B = \frac{3U}{2}\sin(\beta-\alpha)$$

Since potential flow is assumed we may use Bernoulli's equation to relate the pressure, p_A, at A to the pressure, p_B, at B:

$$p_A + \frac{1}{2}\rho u_A^2 = p_B + \frac{1}{2}\rho u_B^2$$

since the effect of gravity is neglected.

Substituting for u_A and u_B:

$$p_A - p_B = \frac{9\rho U^2}{8}\left[(\sin(\beta+\alpha))^2 - (\sin(\beta-\alpha))^2\right]$$

and after manipulation of the trigonometric functions it follows that

$$\sin 2\alpha = -\frac{8(p_A - p_B)}{9\rho U^2 \sin 2\beta}$$