Solution to Problem 109E:

The force transmitted to the automobile through the film of liquid under the tires is given by $\mu Au/h$. Therefore the equation of motion applied to the automobile yields

$$m\frac{du}{dt} = -\frac{\mu Au}{h} \quad \text{and} \quad \frac{1}{u}\frac{du}{dt} = -\frac{\mu A}{mh} \tag{1}$$

and therefore

$$\ln u(t) = -\frac{\mu A}{mh}t + C \tag{2}$$

where C is an integration constant. Denoting the initial velocity at $t = 0$ by U it follows that $C = U$ so that

$$\frac{u}{U} = \frac{1}{U}\frac{dx(t)}{dt} = \exp\left\{-\frac{\mu At}{mh}\right\} \tag{3}$$

where $x(t)$ is the distance traveled after time, t. Integrating

$$x(t) = -\frac{mhU}{\mu A}\exp\left\{-\frac{\mu At}{mh}\right\} + C_2 \tag{4}$$

where C_2 is another integration constant. Setting $x(0) = 0$, it follows that $C_2 = -mhU/\mu A$ and therefore the distance L traveled before coming to rest is given by

$$L = \frac{mhU}{\mu A}\left[1 - \exp\left\{-\frac{\mu At}{mh}\right\}\right] \tag{5}$$

The automobile comes to rest as t tends to infinity and this occurs at a distance

$$L \rightarrow \frac{mhU}{\mu A} \tag{6}$$

With $m = 1000kg$, $A = 0.1m$, $h = 0.0001m$, $U = 10m/s$ and $\mu = 0.001kg/m\ s$, this yields $L = 10km$. While this is ridiculous, the answer does demonstrate that automobiles can hydroplane for a long distance.