Solution to Problem 109D:

Consider a control volume of unit thickness normal to the diagram and bounded by the inclined plate surface, the free surface, and two planes perpendicular to the surface a distance dx apart.

Consider the linear momentum theorem for the direction parallel to the plate surface. Since the velocities and pressures on the ends normal to the plate are identical, they make no net contribution to the momentum flux or pressure forces parallel to the plate. Therefore the only contributions come from the component of gravity and the shear stress at the plate surface so that

$$\rho g h \sin\theta \quad = \quad 2C\mu h \tag{1}$$

and so

$$C \quad = \quad g\sin\theta/\nu \tag{2}$$

Now consider the linear momentum theorem for the direction normal to the plate surface. Since the velocities and pressures on the ends normal to the plate are identical, they make no net contribution to the momentum flux or pressure forces normal to the plate. Neither are there any momentum fluxes through the free surface or the plate surface. Therefore the only contributions come from the component of gravity normal to the plate and the pressures at the free surface, p_a, and at the plate, p_p. It follows that

$$\rho g h dx \cos\theta \quad = \quad (p_p - p_a)dx \tag{3}$$

so that

$$p_p \quad = \quad p_a + \rho g h \cos\theta \tag{4}$$