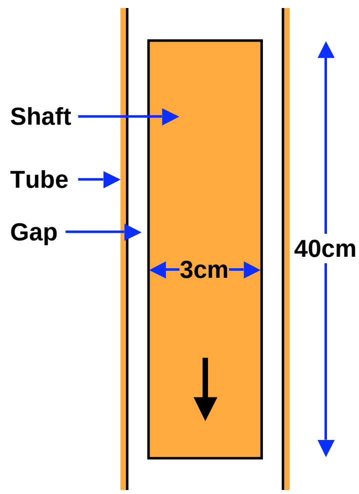
Solution to Problem 109B

The "terminal velocity" is the velocity achieved when all forces on the body balance (i.e. when the body reaches zero



acceleration). In this particular problem, we will neglect the buoyancy force, because the weight of the shaft is much greater than the weight of the displaced fluid. The two forces on the shaft are thus the weight of the shaft itself and the viscous forces acting on the shaft's cylindrical surface:

$$\text{total viscous force} = \text{weight of shaft.}$$

First calculate the weight of the shaft:

$$\text{weight} = (\text{volume})\,(\text{density})\,g = \left(\pi r_s^2 L\right)\rho_s g$$

The shear stress, σ, acting on the cylindrical surface of the shaft is

$$\sigma = \mu \frac{du}{dy} = \mu \frac{u}{h}$$

where u is the shaft velocity and the gap h is

$$h = r_i - r_s = \frac{3.02}{2} - \frac{3.0}{2} = 0.01 \text{ cm} = 10^{-4}\text{m}$$

Then since the viscous force is simply this shear stress multiplied by the area of the cylindrical surface of the shaft it follows that

$$u = \frac{\sigma h}{\mu} = \frac{\rho_s r_s g h}{2(0.9)\rho_w \nu}, = \frac{(0.15\text{ m})(7850\text{ kg/m}^3)(9.8\text{ m/s})(10^{-4}\text{ m})}{2(0.9)(1000\text{ kg/m}^3)(0.005\text{ m}^2/\text{s})}$$

Therefore the terminal velocity is

$$u = 0.0128\ m/s\ = 1.28\ cm/s$$