Solution to Problem 108A

Denoting the atmospheric pressure by p_{atm} and the pressure in the water just below the interface by p_{water} it follows that:

$$p_{water} = p_{atm} - \frac{2S\cos\theta}{r}$$

where, in addition, the hydrostatic gradient of pressure in the water means that

$$p_{atm} = p_{water} + \rho g h$$

Thus,

$$\rho g h = \frac{2S\cos\theta}{r} \quad \Longrightarrow \quad h = \frac{2S\cos\theta}{\rho g r} = \frac{2 \times 0.07 \times \cos 15^o}{1000 \times 9.8 \times 5 \times 10^{-3}} \; N/m^2$$

So

$$h = 2.76 \times 10^{-2} \; m$$

The smallest pressure that the liquid can withstand without vaporizing is 0.017 atm. When p_{water} is equal to this critical values it follows that the maximum height, $h = h_{max}$, is given by:

$$\rho g h_{max} = (1 - 0.017) \times 1.013 \times 10^5 \; N/m^2 \quad \Longrightarrow \quad h_{max} = 10.15 \; m$$

The radius necessary is obtained from the force balance across the interface is:

$$r = \frac{2S\cos\theta}{\rho g h_{max}} = 1.36 \times 10^{-6} \; m \quad \Longrightarrow \quad \text{Diameter} = 2.72 \times 10^{-6} \; m$$