Solution to Problem 100B:

The slope of the saturated liquid/vapor boundary in the pressure/temperature phase diagram is given by

$$\frac{dp}{dT} = \frac{s_{vapor} - s_{liquid}}{\rho_{vapor}^{-1} - \rho_{liquid}^{-1}} \tag{1}$$

where s is the specific entropy and ρ is the density. The above can be obtained using Gibb's potential since that is unchanged during phase transition that occurs at constnat pressure and temperature. It can also be obtained using Maxwell's equation

$$\left[\frac{\partial p}{\partial T}\right]_{constant\rho} = \left[\frac{\partial s}{\partial(1/\rho)}\right]_{constantT} \tag{2}$$

Then by defining the latent heat of vaporization as

$$\Delta h_{vaporization} = T\Delta s_{vaporization} + \frac{1}{\rho}\Delta p \tag{3}$$

where $\Delta p = 0$ so that

$$\Delta s_{vaporization} = \frac{\Delta h_{vaporization}}{T} \tag{4}$$

Thus we obtain

$$\left[\frac{dp}{dT}\right]_{phaseboundary} = \frac{\Delta h_{vaporization}}{TDelta(1/\rho)} \tag{5}$$

This is called the Clapeyron equation.

Since the metastable state (p, T) lies close to the saturated vapor/liquid line it follows that for small ΔT, Δp these quantities can be linearly related by

$$\frac{\Delta p}{\Delta T} \approx \left[\frac{dp}{dT}\right]_{saturatedline} = \frac{\Delta h_{vaporization}}{TDelta(1/\rho)} \tag{6}$$

In water at $100°C$ ($373.15°K$) we have

$$\Delta h_{vaporization} = h_{vapor} - h_{liquid} = (2676 - 419) = 2257 \quad kJ/kg \tag{7}$$

and

$$\Delta(1/\rho)_{vaporization} = (1/\rho)_{vapor} - (1/\rho)_{liquid} = 1.672 \quad m^3/kg \tag{8}$$

so that

$$\frac{\Delta p}{\Delta T} = \frac{\Delta h_{vaporization}}{TDelta(1/\rho)} = \frac{2257}{373.15 \times 1.672} = 3.617 kPa/\circ K \tag{9}$$

thus for $\Delta T = 5°K$:

$$\Delta p = 3.617 \times 5 = 18.1\ kPa \tag{10}$$

Therefore a superheat of $5°C$ corresponds to a tension of $18.1\ kPa$.