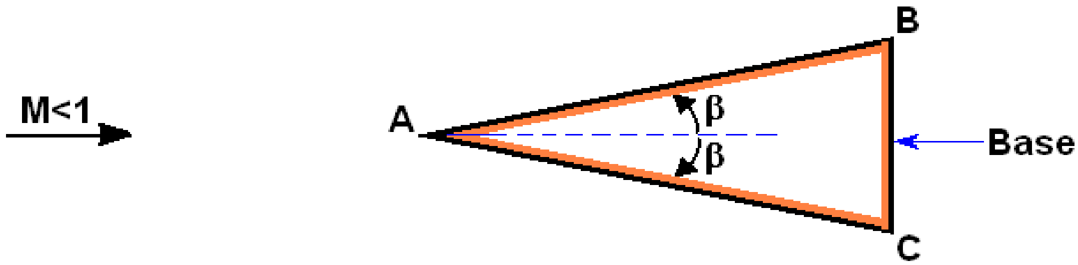
Solution to Problem 345A:

Find the drag coefficient, C_D, for a slender two-dimensional wedge placed symmetrically in a subsonic, inviscid uniform stream of Mach number, M:



The solution to this problem is to find the drag coefficient for the incompressible flow first and then use the Prandtl-Glauert transformation to find the subsonic flow drag coefficient.

First to find the drag coefficient for the incompressible flow. Since $q = Cs^m$ along the inclined surfaces of the wedge, it follows from Bernoulli's equation that the pressure

$$p = p_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho C^2 s^{2m} \tag{1}$$

$$p = p_\infty + \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho C^2 \frac{y^{2m}}{(\sin\beta)^{2m}} \tag{2}$$

where y is a coordinate measured perpendicular to the centerline of the wedge and $y = \pm h$ at the vertices B and C. But since $p = p_\infty$ at $y = h$ it follows that

$$\frac{C^2}{(\sin\beta)^{2m}} = \frac{U_\infty^2}{h^{2m}} \tag{3}$$

and

$$p = p_\infty + \frac{1}{2}\rho U_\infty^2 \left[1 - \left(\frac{y}{h}\right)^{2m}\right] \tag{4}$$

so the coefficient of pressure, C_p^*, is

$$C_p^* = \left[1 - \left(\frac{y}{h}\right)^{2m}\right] \tag{5}$$

Since the pressure on the base is p_∞, the drag per unit depth normal to the sketch, D, is

$$D = 2\int_0^h (p - p_\infty)\, dy = \rho U_\infty^2 h \frac{2m}{(2m+1)} \tag{6}$$

and therefore the drag coefficient is

$$C_D = \frac{2m}{(2m+1)} \tag{7}$$

Then using the Prandtl-Glauert transformation the drag coefficient in subsonic flow is

$$C_D \;=\; \frac{2m}{(2m+1)(1-M^2)^{1/2}} \;=\; \frac{2\beta}{(\pi+\beta)(1-M^2)^{1/2}} \tag{8}$$

Addendum: The above is the most accurate solution to the question. There is, however, another, less accurate solution based on a slender body approach in which the velocity, u, only deviates by a small amount from the free stream velocity, U_∞. Specifically, $u = U_\infty + \Delta u$ where $\Delta u \ll U_\infty$. Then the coefficient of pressure, C_p^*, for the incompressible flow is given by $C_p^* \approx -2\Delta u/U_\infty$ and therefore in the present subsonic flow

$$C_p^* \;=\; 2\left(1 - \frac{Cs^m}{U_\infty}\right) \;=\; 2\left[1 - \left(\frac{y}{h}\right)^m\right] \tag{9}$$

Note that this is different from the preceding solution. However, they both tend to the same limit when Δu and C_p^* are small. However with this alternative C_p^*

$$C_D \;=\; \frac{2m}{(m+1)(1-M^2)^{1/2}} \;=\; \frac{2\beta}{\pi(1-M^2)^{1/2}} \tag{10}$$

which, for small β or m, is approximately the same as the previous result.