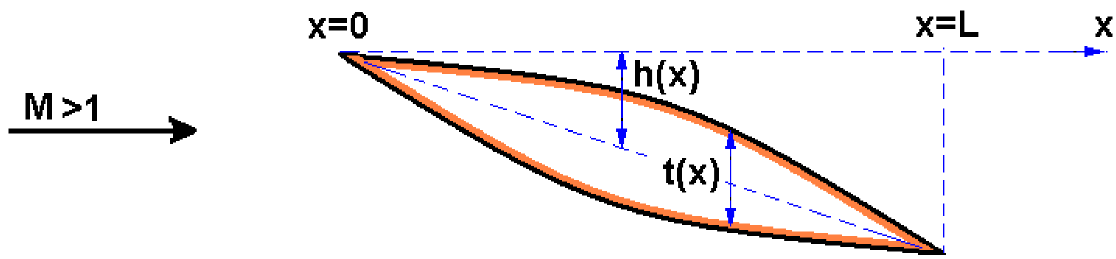
Solution to Problem 340D:



Denoting the pressure and velocity upstream of the foil by p_0 and U then the pressures p_S and p_P on the suction and pressure surfaces are given by

$$C_{pS} = \frac{2(p_S - p_0)}{\rho U^2} = \frac{2}{(M^2-1)^{1/2}} \frac{d(h-t/2)}{dx} \quad \text{and} \quad C_{pP} = \frac{2(p_P - p_0)}{\rho U^2} = \frac{2}{(M^2-1)^{1/2}} \frac{d(h+t/2)}{dx} \tag{1}$$

To evaluate the forces on an increment of the foil length, dx, note that the contributions to the lift and drag from the pressure on the suction surface are respectively

$$-(p_S - p_0)dx \quad \text{and} \quad (p_S - p_0)\frac{d(h-t/2)}{dx}\, dx \tag{2}$$

and the contributions to the lift and drag from the pressure on the pressure surface are respectively

$$(p_P - p_0)dx \quad \text{and} \quad (p_P - p_0)\frac{d(h+t/2)}{dx}\, dx \tag{3}$$

It follows that the lift per unit span, $\mathcal{L}$, is given by

$$\mathcal{L} = \int_0^L (p_P - p_0) - (p_S - p_0)dx \tag{4}$$

and therefore the lift coefficient, $C_L = 2\mathcal{L}/\rho U^2 L$, is

$$C_L = \frac{1}{L}\int_0^L (C_{pP} - C_{pS}) = \frac{2}{L(M^2-1)^{1/2}} \int_0^L 2\frac{dh}{dx}\, dx = \frac{4\alpha}{(M^2-1)^{1/2}} \tag{5}$$

Note that this is identical to the C_L for a flat plate or any foil without thickness.

The drag per unit span, D, is given by

$$D = \int_0^L (p_P - p_0)\frac{d(h+t/2)}{dx} - (p_S - p_0)\frac{d(h-t/2)}{dx}\, dx \tag{6}$$

and therefore the drag coefficient, $C_D = 2D/\rho U^2 L$, is

$$C_D = \frac{2}{L(M^2-1)^{1/2}} \int_0^L \left(\frac{d(h+t/2)}{dx}\right)^2 + \left(\frac{d(h-t/2)}{dx}\right)^2\, dx \tag{7}$$

$$C_D = \frac{2}{L(M^2-1)^{1/2}} \int_0^L 2\left(\frac{dh}{dx}\right)^2 + \frac{1}{2}\left(\frac{dt}{dx}\right)^2 \; dx \tag{8}$$

Then if $dh/dx = \text{constant} = \alpha$ and if $t = 4t_M x(L-x)/L^2$ we can integrate to find

$$C_D = \frac{4\alpha^2}{(M^2-1)^{1/2}} + \frac{16}{3(M^2-1)^{1/2}}\left(\frac{t_M}{L}\right)^2 \tag{9}$$

Note that the first term is the same as that for a flat plate and that thickness increases the drag.