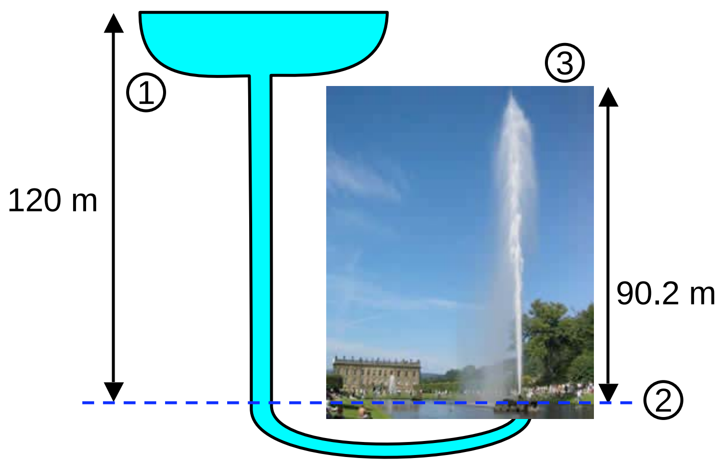
Solution to Problem 274B



1. Taking point 2 as our zero height, Bernoulli's equation shows that the difference in pressure between points 1 and 2 is

$$p_1 + \rho g h_1 = p_2 + \frac{1}{2}V_2^2$$
$$p_1 - p_2 = \frac{1}{2}\rho V_2^2 - \rho g h_1$$

where the velocity at point 2, V_2 is an unknown velocity of the jet (not the velocity in the pipe). Since the pressure at points 2 and 3 are the same (atmospheric) Bernoulli's equation can be used to find the unknown velocity term.

$$p_2 + \frac{1}{2}V_2^2 = p_3 + \rho g h_3$$
$$\frac{1}{2}V_2^2 = \rho g h_3$$

Thus the pressure loss in the pipe is given by

$$p_1 - p_2 = \rho g(h_3 - h_1)$$

Assuming all of this loss occurs in the supply pipe it follows that the friction factor, f, in the pipe is

$$\begin{aligned} f &= \frac{D\left(-\frac{dp}{dx}\right)}{\frac{1}{2}\rho V^2} \\ &= \frac{D\left(\frac{\rho g(h_1-h_3)}{L}\right)}{\frac{1}{2}\rho V^2} \\ &= \frac{2Dg(h_1-h_3)}{LV^2} \\ &= \frac{2(0.381\ m)(9.81\ m/s^2)(120\ m - 90.2\ m)}{(800\ m)V^2} \end{aligned}$$

where V is the velocity of the flow in the pipe. Since the pipe cross-sectional area is $0.114\ m^2$, $1500\ l/min$ means a velocity, $V = 2.2\ m/s$. Therefore $f = 0.058$.

2. The Reynolds number of the flow in the pipe is

$$
\begin{aligned}
Re_D &= \frac{VD}{\nu} \\
&= \frac{(2.2\ m/s)(0.381\ m)}{1.16 \times 10^{-6}\ m^2/s} \\
&= 7.2 \times 10^5
\end{aligned}
$$

3. For pipes, flows with a Reynolds number less than about 2000 are laminar and above 4000 are turbulent. With this Reynolds number, the flow is turbulent. Moreover, referring to the Moody chart, the pipe flow is also fully rough since, at this Reynolds number, the friction factor is well above that for smooth-walled turbulent flow.

4. Again, referring to the Moody chart it would appear that, at this Reynolds number, a friction factor of 0.058 will occur when the roughness has a typical height of $0.03 \times D$ or $1.1\ cm$.

5. With the same friction factor but a pipe diameter of $0.19\ m$ the head loss would be double that of the actual pipe. The maximum height of the fountain would have been $120\ m - 2 \times 29.8\ m$ or $60\ m$ – much less impressive.